DIFFERENTIAL OPTICAL MOTION AND ITS FUNCTIONALITY FOR THE PERCEPTION AND CONTROL OF HEADING

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ABSTRACT

The present program of investigation is concerned with the optical support for the perception and control of heading during constant-altitude approach to a stationary target. Of particular interest is the functional contribution of *differential optical motion*. The results of four experiments in which participants were given control over the direction of computer-simulated self-motion are reported. Results of Experiment 1 served to establish the potential performance range in terms of heading error. In addition, they provided a first indication of the functionality of *motion parallax* for the experimental task and demonstrated that *target drift* can be used for accurate approach. Results of Experiments 2 and 4 showed that successful target approach is not possible in the absence of both target drift and differential optical motion. Furthermore, they served to eliminate two alternative potentially functional types of information (rate of target expansion and apparent rotation of target), as well as provide a first indication of optimizing performance in the top end of the global optical flow velocity range commonly available during human bipedal self-motion. The latter result was replicated in Experiment 3. Experiments 3 and 4 were specifically designed to evaluate the functionality of simple motion parallax (SMP) and differential motion parallax (DMP). A *separation ratio* (σ) indexing the separation of two objects in depth was able to account for (a) performance improvements with decreasing distance to the target, and (b) most of the performance differences among all simulated environments. With the effect of σ accounted for, the addition of DMP information to events that already carried SMP information did not affect performance. The rate of change in horizontal optical separation between at

least two discontinuities is identified as the most likely candidate for the optical foundation of the perception and control of heading during target approach. In conclusion, suggestions for the development of formal descriptions of this variable are made.

CHAPTER I

INTRODUCTION

Ordinarily, we manage to move successfully through our environment without colliding with surrounding objects or other people. What informs us about where we are going? What tells us which way we are heading, where we should be heading, where we should not be heading, and how do we acquire and use this information? The initial answer 'I can see where I am going' does little more than point to the obvious: Much of the information that we need to successfully control the direction of self-motion in our cluttered environment is acquired through vision. As we move around, our retinas encounter patterned arrays of light that transform in lawful ways. One interpretation holds that, since the quality of these 'images' is relatively poor, the brain needs to elaborate/embellish them to enable us to perceive the world as clearly and sharply as we obviously do, and extract the needed information by one or more additional processes of computation (see Michaels and Carello (1981) and Bruce and Green (1990) for a discussion of these and related issues). However, if this is the case, then animals that cannot form a focused 'image' because they lack a physiologically continuous retina should find it difficult to successfully move around using the sense of vision. Insects, having single receptor cells located at the end of cone shaped receptor organs, are an example in case. Nevertheless, they are extremely accomplished in visually guided obstacle avoidance and target approach. Clearly, an alternative explanation is needed.

A second approach holds that we are directly sensitive to higher-order variables that are contained in the transforming *optic* array, i.e., in the pattern of light along a path of observation. The idea that patterns of angular velocities of rays of light carry information about depth relationships was first suggested by Helmholtz (1925). The concept of this 'optic flow' as the principal carrier of information about surface layout and observer motion was pioneered by Gibson (1947, 1950, 1966, 1979). Since then, a number of formal analyses of flow patterns have been carried out (e.g., Gibson, Olum & Rosenblatt, 1955; Koenderink, 1986; Koenderink & van Doorn, 1981; Nakayama & Loomis, 1974; for a review see Heeger & Jepson, 1992). As a well documented example of sensitivity to certain aspects of optic flow, consider the flow pattern produced by approach to an object. Research has shown that, in judging and controlling approach to an object, people as well as animals are sensitive to the inverse of the relative optical expansion velocity, i.e., to the time to contact (Lee, 1976; Lee & Lishman, 1977; Kugler & Turvey, 1987). It is important to note that the focus shifted from trying to understand how we extract information from a succession of discrete retinal images to attempting to answer the questions of (a) what information is contained in the continuously transforming optic array independent of an observer, and (b) what kinds of information are effectively enabling the performance of specific tasks.

I. OPTIC VERSUS RETINAL FLOW

The optic array is here defined as the pattern of light available at single location in space. Optic flow is defined as a temporal change in the structure of the optic array along a possible path of observation prior to the introduction of an eye

(Gibson, 1966). Gibson (1979) distinguished between the *invariant structure* and the *perspective structure* of optic flow. While the former specifies the *layout* of a rigid world, the latter specifies *self-motion* through that world. In addition to specifying the fact of self-motion in general, a particular instance of flowing perspective structure specifies a particular path of self-motion through the environment. Optic flow may be graphically or computationally represented as an instantaneous velocity field, i.e., a temporal mapping of individual optical elements onto a projection surface, where the lengths of the vectors associated with the individual elements correspond to their optical velocity. In the case of linear self-motion parallel to a ground surface this structure may be represented in two dimensions as a pattern of expanding velocity vectors centered about the aimpoint (Figure 1, top panel). Similarly, the pattern represented in the bottom panel of Figure 1 specifies circular self-motion parallel to a ground plane.

Recognition of the fact that what is finally encountered by the retina (in terms of velocity vectors and flow patterns) is influenced by head and eye movements, and consequently bears little resemblance to the optic flow patterns, has led to a considerable number of retinal flow studies (e.g., Cutting, 1986; Cutting, Springer, Braren & Johnson, 1992; W. H. Warren & Hannon, 1988; W. H. Warren, Mestre, Blackwell & Morris, 1991). Retinal flow is formally defined as the changing pattern of light intensities focused on a retina (W. H. Warren, Morris & Kalish, 1988). Consequently, retinal, but not optical, flow is influenced by eye movements. However, it can be argued that these studies are conceptually flawed. As an illustration consider the fact that the instantaneous velocity field of an *optic* flow pattern when the observer undergoes curvilinear translation can be mathematically identical to a *retinal* flow pattern that is encountered when the observer undergoes

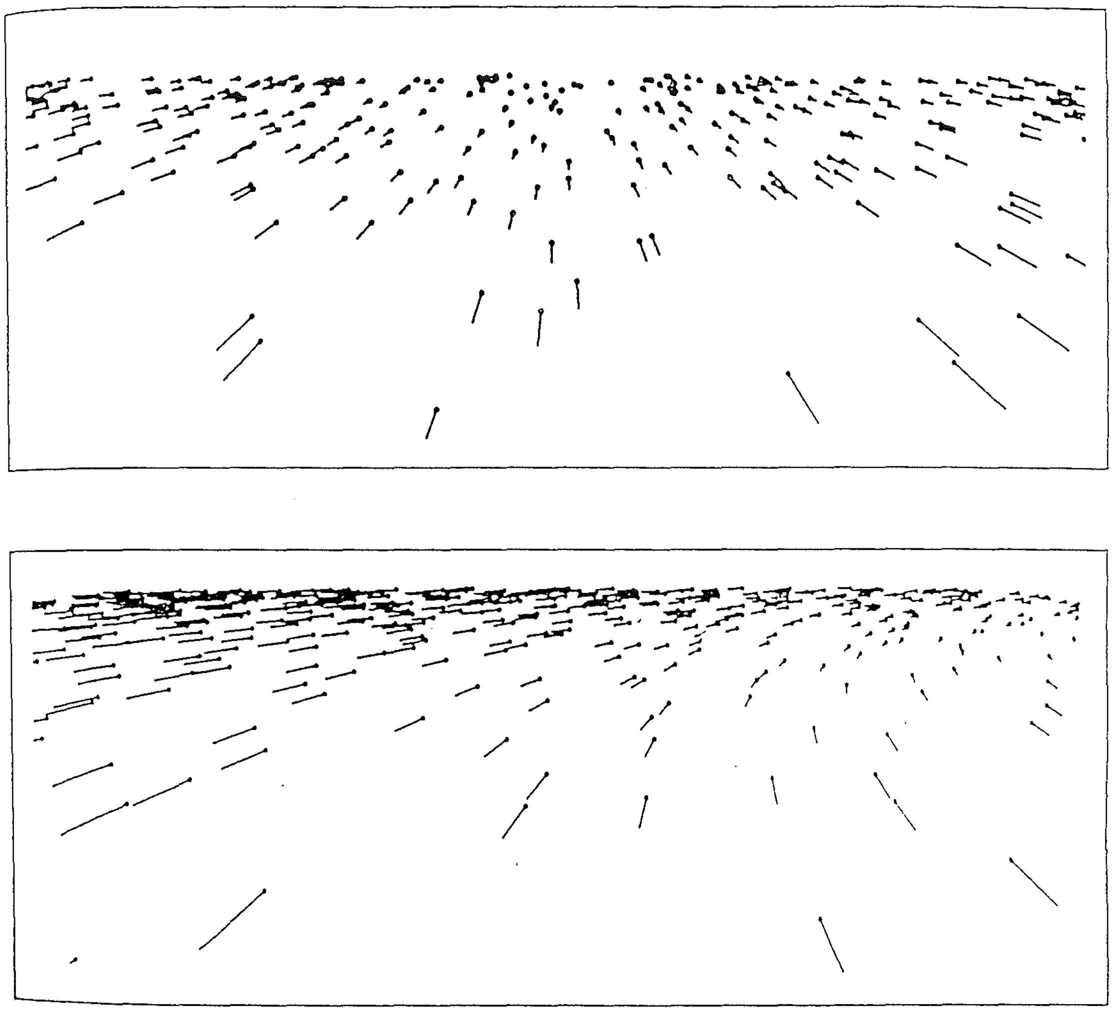

Figure 1. Two-dimensional representation of the instantaneous velocity field of the optic flow generated by observer movement across an endless plane (Top panel: Linear translation parallel to a plane. Bottom panel: Circular translation parallel to a plane) (from W. H. Warren, Mestre et al., 1991).

linear translation, but fixates an optical element to one side of the movement path (W. H. Warren, Blackwell, Kurtz, Hatsopoulos & Kalish, 1991; W. H. Warren, Mestre et al., 1991). In both cases the retina encounters the same velocity field, yet in the former perception is of heading around a bend, whereas in the latter it is of heading straight ahead while looking off to the side of a linear path of movement. If this is the case, the question that has to be addressed is how we can perceptually distinguish between two events with identical retinal flow fields. One explanation holds that we engage in some form of computational process that allows us to decompose the retinal flow field into rotational and translational components. This assumption would lead to a search for the processes by which this is achieved. Indeed, researchers have already developed a number of computational models that provide potential solutions to this problem of *visual decomposition* (Bruss & Horn, 1983; Longuet-Higgins, 1984; Longuet-Higgins & Prazdny, 1980; Nagel, 1981; Prazdny, 1980; Prazdny, 1981; Rieger & Lawton, 1985; Tsai & Huang, 1981; Waxman & Ullman, 1985). However, this explanation is based on an implicit acceptance of a mediational theory of perception, i.e., of the idea that we are dealing with a representation of the pictures / images / velocity fields that are projected onto the retina. An alternative explanation simply holds that perception is anchored to the optic array in front of the eye, not on the back of the eye. We do not 'see' the retinal flow. Instead, perception has evolved to be sensitive to optic flow only. Eye movements simply serve to make available information contained in the optic array by allowing the perceptual system to sample different sectors of the optic flow pattern. W. H. Warren and Hannon (1990) point out that if the flow field is permitted to unfold over time, any ambiguities are resolved. In addition, extra-retinal information (proprioceptive feedback as a result of head and eye movements) may serve to further specify the difference (Rock, 1968; Royden, 1994; Royden, Banks & Crowell, 1992[1]; Royden, Crowell & Banks, 1994). Consequently, we do not confuse one event with another, even if the retinal velocity fields are (briefly) identical. If this is accepted, the focus shifts once more from trying to understand a non-existent process of decomposition to attempting to achieve a more complete understanding of the informative variables and invariants that are contained in the transforming optic array.

II. OPTICAL SUPPORT FOR THE PERCEPTION OF HEADING

Research on way finding and the perception of heading originated with Gibson's (1947, 1950) demonstration that translation of an observer through a stationary environment produces a radial pattern of optic outflow at the aim point, i.e., at the point in the transforming optic array toward which an observer is heading[2]. This resulting 'focus of expansion'[3] is specific to the observer's direction of movement, while the movement path is linear (W. H. Warren, Mestre et al., 1991). Since then, a number of researchers have attempted to identify and isolate additional

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[1] For a reply see van den Berg (1994).

[2] The term 'heading' will be used to denote the instantaneous direction of self-motion. In contrast, to denote the direction in which one is facing, the term 'viewing direction' will be used.

[3] Since the actual rate of expansion is zero at that point (Koenderink & van Doorn, 1981), and the term 'focus of expansion' is consequently a misnomer, Gibson's (1979) later term, 'focus of radial outflow' will be adopted.

functional types of information[4] in the transforming optic array that specify heading for both linear and curvilinear translations.

(1) Information for Linear Translation

As an observer moves along a linear path, the optic flow field is characterized by a radial pattern of outflow centered about the aimpoint, and a radial pattern of inflow in the direction from which an observer is moving. Gibson's (1947) original formulation of global radial outflow states that the direction of heading is specified by the *global* pattern of the transforming optic array. Note that, unlike the *magnitude* of the velocity vectors in the optic array, their *direction* is completely dependent on the direction of observer movement and does not vary with surface layout (Gibson, 1947; Nakayama & Loomis, 1974; Prazdny, 1981). In other words, while an optical element produced by a near object flows faster than one produced by a far object, the direction of the flow is completely dependent on the direction of self-motion. Since this transformation affects all elements of the array, locating the single stationary point coinciding with the focus of radial outflow (FRO) is not necessary. However, if this is at all possible, this "local focus of outflow" (W. H. Warren et al., 1988, p. 647) may be an important source of information. For example R. Warren (1976) showed that performance on a pointing task in response to simulated linear motion across an endless plane improves when the FRO is visible. It might be found by detecting the motion of a fixated element and subsequently shifting the gaze in the direction opposite to its movement. In addition, peripheral vision may suffice to locate the approximate location of that point (Crowell & Banks, 1993). Generally speaking, a

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[4] See Owen (1990) for a useful classification of the functionality of information.

variety of computational methods can be used to determine its location by calculating the intersection of a given set of flow vectors (cf. Lawton, 1983; Prazdny, 1981). Supporting neurophysiological evidence comes from a study by Saito, Yukie, Tanaka, Hikosaka, Fukada and Iwai (1986) who found a class of cortical cells in the superior temporal sulcus of the macaque that responds selectively to expanding or contracting patterns. Similarly, Regan and Beverley (1979) argue that the localization of a flow pattern's focus could be facilitated by 'channels' that respond to changes in the size of small objects.

On the other hand, existing psychophysical evidence suggests that human observers may not be capable of locating the FRO when approaching a surface oriented in the fronto-parallel plane. For example, using a half-dome projection, Johnston, White and Cumming (1973) demonstrated that in such conditions, observers are only able to locate the FRO with an accuracy of approximately 10 deg of visual angle. In contrast, accuracy of heading judgments in studies employing simulations of discontinuities at varying depths is typically around 1 to 5 deg (e.g., van den Berg, 1992; Cutting et al., 1992; W. H. Warren & Hannon, 1988; W. H. Warren & Hannon, 1990; W. H. Warren et al., 1988). Some additional information in terms of differential motion of optical elements generated by objects located at varying distances from the moving point of observation seems necessary. In addition, since a stable FRO is only available during linear self-motion, its general usefulness may be questioned (W. H. Warren, Mestre et al., 1991). Finally, considerations of retinal flow fields yielded the observation that the FRO does not uniquely specify the direction of self-motion. Consequently, a number of authors rejected the FRO as useless for the perception of heading, and instead proposed a variety of alternative types of information. W. H. Warren and Hannon (1990) presented a brief but comprehensive overview of such analyses, and interpreted subsequent tests of each of the models to indicate that differential motion between neighbouring optical elements (Rieger & Lawton, 1985) is "both necessary and sufficient for a decomposition based on flow-field information" (p. 168).

In contrast, Cutting (1986; Cutting et al., 1992) disagreed with the idea that humans decompose retinal flow into translational and rotational components to derive heading information. However, rather than presenting an argument for human sensitivity to *optic* flow, he proposed that information contained in *retinal* flow is directly used to perform this task. More specifically, the relative motion across the retina of far and near objects against a fixated object in the middle distance is proposed to be used to determine the heading (see also Priest & Cutting, 1985). This represents an elaboration of the idea that the differential optical motion of at least two elements in a transforming array (motion parallax), via triangulation of the optical velocities of those elements, may be used to determine translational heading (Longuet-Higgins & Prazdny, 1980; Rieger & Lawton, 1985). According to Cutting's *differential motion parallax* (DMP) hypothesis, the highest retinal velocity across the line of fixation is in a direction opposite to that of observer movement. The magnitude of that velocity vector is directly proportional to the deviation of the aimpoint from the direction of the gaze, and can consequently be used to determine the direction of self-motion.

(2) Information for Curvilinear Translation

Self-motion along a curvilinear path introduces a rotational component into the optic flow field. A circular movement path can be formally described as the sum of a rotation around a vertical axis through the eye and a translation along the tangent to that path. If the radius is constant, the resulting flow field generated by a ground plane will have the appearance of concentric circles (see Figure 1, bottom panel). The velocity field of the optic flow in the direction of travel takes on the appearance of a family of hyperbolae.

In the case of circular or curvilinear translation there is no stable focus of radial outflow available in the optic array. W. H. Warren, Mestre et al. (1991) were able to show that people, when viewing random-dot optic flow displays simulating motion along a circular path, are able to judge heading with relatively high and constant accuracy over a wide range of dot densities (from 3.3 deg error in the minimum condition of 2 dots to 2.5 deg in the maximum condition of 62 dots). The authors interpreted these findings to support the notion that vector normals, rather than two alternative potentially functional variables of the optic array (locomotor flow line and reversal boundary) are utilized to determine heading[5]. A vector normal is the geometric normal to the velocity vector of a given optical element. If the vector normals of such an element in two successive samples are parallel, translation is linear. If translation is circular, the two vector normals intersect at the center of rotation, thereby specifying the path of translation. However, unless all curvilinear, but non-circular, paths are viewed as variations of circular paths, it remains unclear how this type of information behaves.

An alternative formulation implicates information made available by DMP (Cutting, 1986; Cutting et al., 1992). This type of information has the advantage of

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[5] Extraction of information from both locomotor flow line and the reversal boundary requires a reasonably dense flow field.

specifying heading independent of the form of the path of translation. In the following section this idea is presented in more detail.

(3) Motion Parallax

The term 'parallax' has its origin in astronomy, and denotes the apparent displacement of a distant object due to a change in position of the observer. The term 'motion parallax' is more commonly used to denote the (horizontal) optical motion of a discontinuity produced by a near object against that produced by a far object. Helmholtz (1925) first noted that these angular velocities are informative about the distance between the observer and the objects. Gibson et al. (1954) pointed out that they "carry information not only about the distances of objects but also about the direction in which [an observer] is moving" (p. 374). In the present investigation the term 'simple motion parallax' will be used to denote the differential optical motion produced by two objects at different distances to the moving observer, and the term 'differential motion parallax' to denote the differential optical motion produced by at least three such objects.

(a) Differential Motion Parallax and Optic Flow. To reiterate, Cutting's (1986) differential motion parallax (DMP) hypothesis takes the retinal flow pattern as the point of departure for the acquisition of information for the perception and control of heading. Accordingly, during linear translation, when an observer looks exactly in the direction of translation, retinal flow in the horizontal dimension can be described as a function of change in z, by the formula

$$d\theta/dz = -x/(x^2 + z^2), \qquad (1)$$

where z is the axis extending in the direction of self-motion, x is the axis extending laterally at right angles to z, and θ is the horizontal optical angle between the direction of translation and a given optical element. This equation is graphically represented in Figure 2. Note that it also describes optic flow, i.e., the transformation in the patterned array of light at a possible moving point of observation *prior* to the introduction of the eye. All velocity vectors in the array point away from the direction of self-motion, with elements located on the same isoangular displacement contour possessing identical vector lengths.

As the moving observer fixates an object in the environment that does not exactly lie in the direction of self-motion, Cutting (1986) shows that the isoangular displacement contours of the retinal flow change such that they become asymmetrical about the line of movement. If an object to the right of the direction of self-motion is fixated, the circular arrangement of the displacement contours on that side of the direction of self-motion becomes smaller compared to the one on the opposite side. However, he fails to show that this is essential for differential motion parallax information to become available at the point of observation. Essentially, he shows that if an observer were to fixate an object on the contour with isoangular displacement of .4 (Figure 2), the retinal displacement of objects lying on other contours would decrease by exactly that amount (.8 becomes .4, .2 becomes -.2, etc.). Note that in this instance negative numbers indicate retinal motion towards the direction of self-motion, while positive numbers indicate retinal motion away from that direction. The net result is that the most rapid retinal motion is in a direction opposite to that of the direction of self-motion. This will always hold true as long as

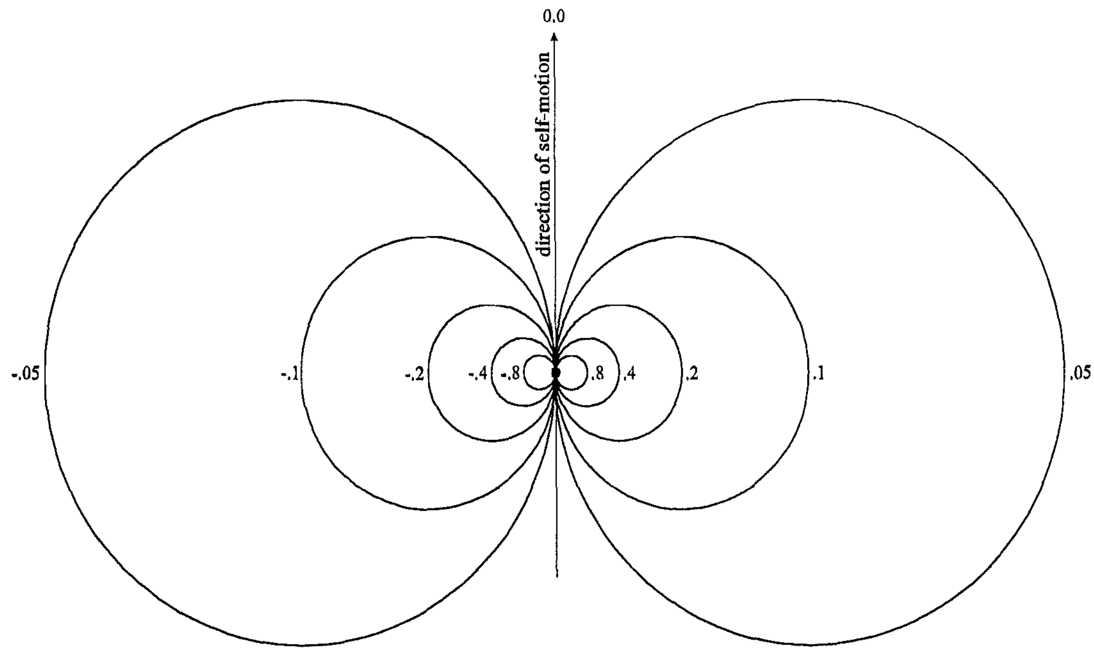

Figure 2. Isoangular displacement contours in the horizontal plane. The moving point of observation (viewed from above) is indicated by the filled circle. All discontinuities produced by objects in the environment located on a given contour flow laterally with the same instantaneous displacement (from Cutting, 1986, p. 195). The numbers are calculated using Equation 1, where z = 0, and x = +/-1.25, +/-2.5, +/-5, +/-10, +/-20, and ∞.

the distance between the moving point of observation and the near object is half or less the distance between the observer and the fixated object. It also holds true when near and far objects are equally distant from the fixated, or middle, object. A sequence of fixations away from that motion would then allow the determination of the direction of self-motion (Cutting, 1986, Cutting, 1991; Cutting et al., 1992). He expresses this relationship more formally as

$$N > -F, \tag{2}$$

where N is the velocity of near elements (note the positive sign), and F that for far ones (Cutting, 1986, p. 219). He terms this relationship an *inequality invariant* in retinal flow, and proposes that it can be used as the basis for the perception and control of heading[6].

In contrast, an argument can be made that this information is also available in optic flow, independent of any eye movements. In this case, however, it makes sense to describe the *relative rate of lateral deviation*. In other words, any object on the contour with isoangular displacement of .4 produces an optical velocity of half that of an object that is exactly half the distance away from the moving point of observation (isoangular displacement contour = .8), and double that of an object that is exactly twice the distance away from the moving point of observation (isoangular displacement contour = .2). As long as the observer is both sensitive to the *relative* differences in optical velocity *and* attends to them, it is immaterial whether or not s/he actually fixates the object in the middle distance. Although more often than not the activity of attending will be accompanied by a fixation, it does not seem to be a necessary condition. The important point is, however, that *the amount and type of information conveyed by retinal flow is identical to that conveyed by optic flow.* To find the direction of self-motion, rather than shifting the gaze in the direction opposite that of the most rapid retinal flow, it suffices to shift it in the direction opposite that of the most rapid optic flow. Notice that for both conceptualisations a certain

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[6] Although his exact words were "inequality invariant in *optic* flow [emphasis added]" (Cutting, 1986, p. 219), the present description is more consistent with the thrust of his argument.

differentiation in 'depth' between the optical elements, i.e., in distance from the moving point of observation, is necessary.

Although the situation becomes mathematically a little more complex with curvilinear self-motion, the same argument can be applied. While Cutting (1986) shows that retinal flow during curvilinear translation contains DMP information that can be perceived and used to control the direction of self-motion, the solution can again be rephrased in terms of relative motion in the optic, rather than the retinal, flow pattern.

Research evidence. Cutting (1986) describes a sequence of experiments in which he simulated approach to three sets of vertical bars all subtending the full height of the display. Each set was arranged along a line perpendicular to the direction of simulated self-motion, and situated at different distances from the point of observation at the onset of a trial. The simulation was centered or 'locked' on a vertical bar of the set in the middle distance, and linear translation was simulated at some angular deviation from that gaze direction (gaze-movement angle). Subjects were told to fixate on that bar and to decide whether they were looking to the right or to the left of the direction of simulated self-motion. Results indicated that participants could correctly determine the direction of self-motion on 80 % of trials with final gaze-movement angles of around 5 deg. Performance decreased with decreasing final gaze-movement angles and with decreasing spatial separation of the three sets of bars. Two experiments simulating circular approach to the three sets of bars yielded comparable results. Cutting et al. (1992) reported an additional sequence of experiments in which they further investigated the potential usefulness of DMP for the task of wayfinding. The same basic experimental manipulation was employed, i.e., pursuit fixation of an object to one side of the path of translation was simulated, and at the end of the simulation a judgment of whether the simulated fixation point was to the right or the left of the direction of self-motion was requested. The displays consisted of an endless plane with either simulated trees constructed of lines that grew longer, but not wider as they were approached, or of disks lying flat on the plane. At a 75 % correct criterion the reported accuracy ranged from 0.6 to 1.8 deg, and at a 95 % criterion from about 1.5 to 8.5 deg. Overall, results were interpreted to indicate that DMP information contained in retinal flow is used in the perception of heading. In contrast, I have argued that DMP information is also available in optic flow. The results obtained be Cutting and colleagues show that DMP information can be used in the perception of heading. Importantly, however, it has not been shown that DMP provides functionality over and above that provided by simple motion parallax.

Note also that Warren and Hannon (1990) have reported results from a passive observation study into heading judgments, which they interpreted to be inconcistant with the DMP hypothesis "because observers succeed with sparse fields and without multiple fixations" (p. 168). However, while multiple fixations may well facilitate the accurate detection of heading information from DMP, there is no reason to assume that they are absolutely necessary.

III. THE CASE FOR AN ACTIVE PSYCHOPHYSICS

Interesting and useful as the above results may be, a good argument can be made against their general ecological validity. Experimental tasks have so far mostly been designed such that observers are presented with a simulation of being transported through an environment, akin to being passengers in a moving vehicle. The simulated viewing direction is typically that encountered when looking through the windscreen of a moving vehicle. Alternatively, pursuit fixation on an object to either side of the movement path is simulated. At the end of the simulation, while the last frame is frozen on the display, a target appears on the display, and a judgment of whether the previously viewed path of translation would pass to the left or the right of that target, or whether the fixation point is to the left or right of the movement path is requested. A staircase procedure is then used to arrive at a criterion correct response (typically around 75 %). This figure is then taken to be indicative of a perceptual threshold for the perception of heading, and performance is evaluated with respect to the information present in the simulation.

This type of analysis is fine with respect to weeding out the most unlikely candidates. When a certain type of information that specifies heading is presented in a simulation, and performance drops below an acceptable level, it is safe to assume that the actors are not attuned to it, i.e., the information is nonfunctional or noninformative (Owen, 1990). A problem arises when trying to differentiate between different functional types of information. The fact that an observer *can* use a particular type of information to perform a task in a very constrained environment and under very constrained conditions of observation, does not necessarily mean that s/he also uses it under different, perhaps more ecologically valid circumstances. The ability of a human observer to use information made available by differential motion (Rieger & Lawton, 1985), vector normals (W. H. Warren et al., 1991), or DMP (Cutting et al., 1992) to judge the direction of a previously viewed path of translation with respect to an object in the simulated environment does not necessarily imply the use of such types of information by a human actor who purposefully moves through that environment.

It is of course not always the case that humans, and much less other animals, find themselves in a position where they have to judge their relative heading in an environment without having active control over the direction of their movement. Although in some professions people spend progressively larger amounts of time as passive observers and are only required to take control in certain situations (e.g., aviation), I concur with Laudeman and Johnson (1993) that a complete understanding of both modes of operation is essential. It is suggested that the study of the perception of heading and wayfinding has been constrained to somewhat limited situations and tasks. As observers actively and purposefully control their movements in their respective environments, they produce and control optic array transformations simply by the act of moving around (cf. Owen, 1990; Owen & R. Warren, 1982; Owen & Wolpert, 1987; R. Warren & McMillan,1984). The study of those movements and actions, and of the information that emerges as a result of them is the topic of the present program of investigation.

(1) Research Evidence

There is evidence to suggest that performance in active and passive observation tasks differs. A number of studies have shown that performance on a variety of tasks depends on whether the events are being actively or passively observed. For example, active observers have shown better performance in tracking tasks in terms of latency and accuracy of failure detection than their passive counterparts (Wickens & Kessel, 1980). More recently, Laudeman and Johnson

(1994) simulated level flight over an endless plane consisting of computer generated dots. The subject's task in the active mode consisted of controlling the direction of motion in the lateral dimension such that a disturbance function was compensated for. The flight paths that a given subject generated were then presented to the same subject in a passive observation mode. At the end of each trial a judgment with regard to the final instantaneous direction of travel was required. Results demonstrated that awareness of the instantaneous direction of motion was better for active than for passive observers. Similar results with regard to awareness of the orientation of an aircraft after flying through cloud cover (simulated by a blank screen) were obtained by Larish and Andersen (1991).

Only a few active control studies of the perception of heading have been conducted to date (Beall & Loomis, 1995a; Beall & Loomis, 1995b; Land & Lee, 1994; Laudeman & Johnson 1993, 1994; Levison & R. Warren, 1984; Owen & Wolpert, 1987). Laudeman and Johnson were interested in a comparison of active versus passive observation, and as a consequence did not take continuous recordings of control inputs to make inferences about optical information or perceptual capability. On the other hand, Beall and Loomis' work represents, along with the present program of investigation, one of the first true active control studies of heading perception. In their first study (1995a) they provided a test of an optic flow rule postulated by Loomis and Beall (1992) that might be used to control the flight path of an aircraft through a turn from base leg to final leg of a landing approach. In simple terms, all a pilot has to do to successfully complete such a manoeuvre is to control the flight path of the aircraft such that the rate of change in splay (the temporal derivative of the optical angle formed between the center line of the runway and a normal to the horizon) remains approximately constant or invariant. While the authors demonstrated that a computer-controlled model can successfully use this rule, their investigation of actual flight paths of three pilots using a global positioning system receiver suggested that human controllers may use a more complex control strategy. Their second investigation (1995b) concerned itself with the relative importance of (a) rate of change in splay (in this case defined as the temporal derivative of the optical angle formed between two parallel lines stretching to the horizon) and (b) motion parallax (here defined as the temporal derivative of the optical angle formed between the viewing direction and a landmark)[7] for the task of keeping to a straight path in the presence of laterally perturbing forces during simulated flight. Results were interpreted to indicate that both motion parallax and rate of change in splay can be used to perform this task successfully. Land and Lee (1994) investigated eye movements of three drivers while they negotiated a narrow one-way road. They showed that the changing optical angular deviation of the tangent or 'reversal point' of a curve from the heading of the vehicle is a very good predictor of the actual curvature of the road ahead, and found that people spend a considerable amount of time looking at that point. Finally, Levison and R. Warren (1984) and Owen and Wolpert (1987) were the first to use an active control paradigm to study the perception and control of altitude during self-motion (a special case of heading perception). In both studies subjects had the task of correcting for a forcing function representing either upward or downward windshear during a simulated flight task. Levison and R. Warren investigated the effects of varying splay angles and levels of display gain on performance. They found that performance improved with

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[7] Note that this is a rather unusual use of the term. A more common term for this type of optical transformation is 'target drift'.

increasing gain and remained relatively constant over the two investigated levels of splay angle (30 and 60 deg). Owen and Wolpert were interested in (a) determining the relative influences of optic flow velocity and edge rate (the rate at which edges flow by) on sensitivity to loss in altitude, and (b) testing the generalizability of results from earlier passive observation studies. Results demonstrated the usefulness of the active control paradigm and indicated that sensitivity to change in altitude deteriorates with increasing flow velocity. In contrast, the impact of varying levels of edge rate was minimal.

(2) Some Potential Problems

R. Warren (1990) suggested that there may be problems associated with the measurement of heading during active control. He argued that the ability to perform a given task, or, in more general terms, to control a given system, depends not only on our ability to perceive deviations from the ideal state, but also on our ability to control the system. Consequently, it is impossible to separate the two. In terms of an analogy, "my *in*ability to hit a bull's eye does not mean that I couldn't *see* the bull's eye" (p. 13). However, he fails to note that the cited example is a case of out-of-loop control, i.e., with release of the dart, the thrower relinquishes control over its trajectory. In contrast, consider another example: The task is to orient a battery driven toy car such that it hits a predetermined target. If the car has a remotely controlled steering system the driver will be able to engage in appropriate control actions as soon as s/he perceives the car to deviate from the ideal heading. Even though levels of ability to control such a system may vary between people and accuracy of control movements may be less than ideal, the angular deviation of the movement path from the target at which control actions are initiated and concluded may be used to draw valid conclusions about the sensitivity of observers to information about *object motion* present in the optic array. To take this one step further, assume that the toy car is fitted with a camera oriented in the direction the vehicle is pointing and that the driver is required to control the heading solely on the basis of the filmed visual information. In this case the control actions and the consequent movement path in relation to the optical information available to the driver become informative about the perception and control of *self-motion* in general, and of heading in particular.

A second caveat raised by R. Warren relates to the fact that an actor may not be aiming for a specific target point, but rather a target neighbourhood. While that in itself seems to be a subject worthy of attention (for example, it seems reasonable to assume that the deviation of the actual movement path from the ideal one decreases proportionally with time to contact), R. Warren's argument relates to the issue of intention and goal directed behavior. It is suggested, that if both the goal and the intention are to reach the target while deviating as little as possible from a predetermined path, rather than to just reach the target, then a study of the control actions and its consequences will most certainly give insights into perceptual ability as well as the functionality of different types of information.

IV. AN INITIAL EXPLORATION

The aims of the present program of investigation are to evaluate the functionality of optic flow information for active perceivers (in particular that of simple and differential motion parallax) and, in the process, to demonstrate the utility of an active control paradigm for the study of the perception and control of heading.

Two criteria were considered important in choosing the experimental task and the simulated environments. (a) The task should be one that participants are familiar with, i.e., one that they encounter in everyday life. (b) The environments should be kept simple as well as ecologically valid, while still allowing precise control of the presented information to evaluate its impact on performance.

To begin with a relatively simple scenario, movement parallel to an endless horizontal plane was simulated. Initial distance to the simulated ground plane was kept constant across trials and velocity relative to it was kept constant within trials. Participants had first-order (velocity) control over the yaw angle and the resulting movement direction via an analog joystick. The low order of control was intended to keep task difficulty low. Gibson (1966) stated that "visually guided locomotion is a matter of going to a specific goal in the environment" (p. 162). Consequently, the experimental task consisted of straight-line approach to a target.

The aims of Experiment 1 were twofold: (a) It was considered necessary to establish the potential performance range of the experimental task. This was intended to show that the addition and subtraction of different types of information to and from displays could have a measurable and practical impact; measurable in terms of statistically significant differences, and practical in terms of effect sizes. For example, if the overall performance range in terms of heading error turns out to be 0.3 deg, say, it may well be possible to find statistically significant differences within that range. At the same time the practical difference of such small differences may well be negligible. The performance limits, once known, can be used to evaluate the relative usefulness of the manipulated optical variables for the task at hand. Consequently, two environments were designed for this purpose. One was intended to allow evaluation of heading thresholds under minimal conditions, the other under optimal conditions. (b) The introduction of an environment that made differential motion parallax information available to the observer was intended to provide a first indication of the potential functionality of that type of information for the perception and control of heading.

CHAPTER II

EXPERIMENT 1

I. METHOD

(1) Participants

The participants were six right handed male graduate students in psychology and computer science at Canterbury University. They were between the ages of 20 and 24 years.

(2) Apparatus

(a) Hardware. Events were generated using an XTAR SuperFalcon AP-4000 floating-point array processor and graphics board. Events were displayed on a Hitachi SuperScan colour display at a frame rate of 60 Hz and a resolution of 1024 by 1024 pixels. The display was 35 cm wide and 28.5 cm high. The host computer was a 486DX2-33 PC.

A pyramidal viewing hood lined with black nonreflective material, ending in a mask fitting around the orbit of the eyes, was used to ensure that (a) perceptual information unrelated to the simulation was minimised, and (b) the binocular viewing distance was kept at the correct geometric projection point of 54.5 cm. Consequently, the projection surface filled visual angles of 29.3 deg (vertically) and 35.6 deg (horizontally).

Events were controlled using a precision analog joystick (12-bit resolution, Measurements Systems, model no. 22978) mounted at the end of the right armrest of a chair.

(b) Kinematics. The experimental task was to control the simulated movement path of the eye such that a previously specified target was reached (see p. 34, for details). Simulated altitude [eyeheight (h)] and global optical flow velocity (GOFV) were kept constant throughout the trials at 500 units and 1.2 h/s, respectively. Targets were located at an initial lateral deviation of 0, 2, or 4 deg to either side of the centre of the display resulting in five possible target locations[8]. The initial (linear) movement path was such that if no control action was engaged in, the target would be passed at 2, 4, or 6 deg to either side of the target. Consequently, the focus of outflow was at 0, 2, 4, 6, 8, or 10 deg from the center of the display for the duration of a trial[9]. This was intended to prevent the strategy of using local sources of information, i.e., of simply keeping the target centred on the display.

The joystick was self-centering with a noticeable center position. As long as it was centered, the simulated self-motion was always linear. A deadband of 5 units was centered on the zero position, such that unintended control movements (e.g., vibrations of the hand) would go unrecorded. Deflections to the left or right resulted in a change in movement direction to the same side. Forward or backward deflections had no effect on the simulation.

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[8] Positional values in degrees are taken to denote the horizontal optical angle delineated by a reference point (in this case the centre of the display) and the object of interest (in this case the target) at the onset of a trial.

[9] A situation like this might be encountered when flying a fixed wing aircraft towards a target under crosswind conditions. This is commonly dealt with by pointing the aircraft into the wind until the target appears stationary with respect to its lateral position in relation to the windscreen. A similar situation might arise in naval navigation when cross currents occur. A more common example would be walking while keeping the head turned to one side. Borrowing from camera terminology this type of simulation has been referred to as 'dolly' technique (Cutting et al., 1992).

The joystick was a first-order (velocity) controller, influencing the rate of change in direction of the simulated self-motion (rad/s). During testing it was found that a gain of 2.1×10^{-4} was small enough to allow accurate control for small adjustments, while still being responsive enough to allow large control adjustments without reaching saturation. To illustrate, a stick deflection to the left resulting in an output of 100 units would be multiplied by 2.1×10^{-4} to yield a rate of change of 0.021 rad/s.

(c) Environments. All environments consisted of an endless gray ground plane meeting with an off-white sky at the vertical midpoint of the projection surface. All simulated objects in the environments were constructed of diamond shaped, vertically linked polygons. This particular form was chosen to minimize aliasing (the sudden displacement of the vertical edge of a slow-moving object due to pixel size), and to make both horizontal and vertical components of optic flow available to the observer. In each environment a target was situated at an initial distance of 24 h from the initial point of observation. With a GOFV of 1.2 h/s the target was passed after approximately 20 s. The target always consisted of two sets of six bright yellow, vertically linked diamonds. It was always positioned such that one of its surfaces was at right angles to a straight line connecting it with the initial point of observation. All other objects in the display consisted of four diamonds coloured in one of three shades of grey. A brief description of each of the three experimental environments follows. For a more detailed description of all objects and environments, refer to Appendix A.

Target-only Environment. The Target-only environment consisted of ground plane, sky, and target. This environment was intended to function as a base-line control by allowing evaluation of heading thresholds under minimal conditions. The only functional optical information present in the simulation consisted of the expansion of the target, and 'target drift', i.e., the apparent lateral movement of the target across the display in a direction away from the focus of radial outflow (Figure 3).

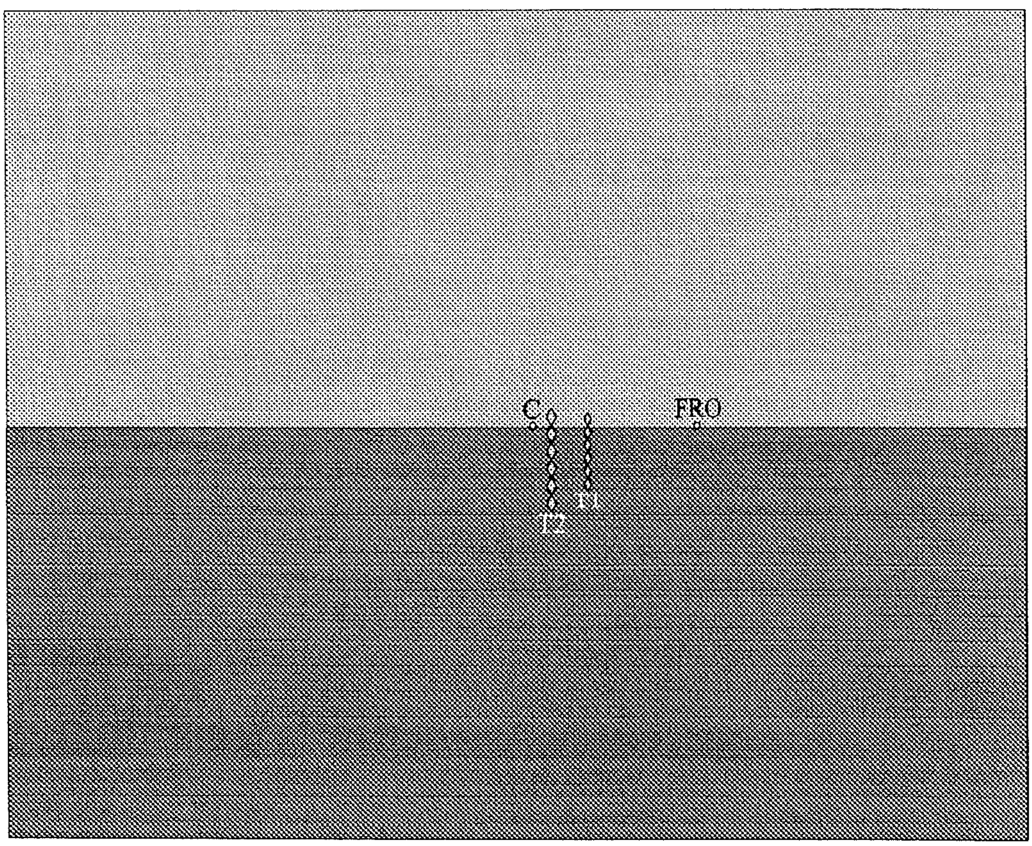

Figure 3. Target-only environment. C = centre of display, T1 = target at trial onset (initial deviation of target from centre of display = 2 deg), FRO = focus of radial outflow (initial deviation of FRO from target (initial heading error) = 4 deg), T2 = target after 5 s at a global optical flow velocity of 1.2 h/s.

Differential Motion Parallax Environment. In this environment enough information was included to ensure that differential motion parallax information would become available. To achieve this, six rows of objects were added to the Target-only environment, such that three rows were placed on each side of, and parallel to, a straight line connecting the initial point of observation and the target (Figure 4). Objects to the (left) right of that line were rotated (anti)clockwise 20 deg about the vertical axis, thus maximising the visual angles enclosed by them as the simulated egolocus, i.e., the simulated moving point of observation, moved past. If the shortest possible route was taken to reach the target, the layout of the objects to either side of the movement path ensured that differential motion parallax information became available for the first time after 2 h of simulated self-motion towards the target and after that every 4 h. This information was available along any 'plane of sight' that intersected any object (other than the target) in the simulated environment. The term 'plane of sight' is used in preference to the more commonly used term 'line of sight'. Strictly speaking, a line of sight only refers to a single point in the optic array. Consequently, if two opaque objects cross a line of sight at the same time, one must necessarily occlude the other. In contrast, the term 'plane of sight' refers to a vertical slice of the optic array, i.e., a vertical plane viewed side-on. If two opaque objects cross a plane of sight at the same time, occlusion may or may not occur.

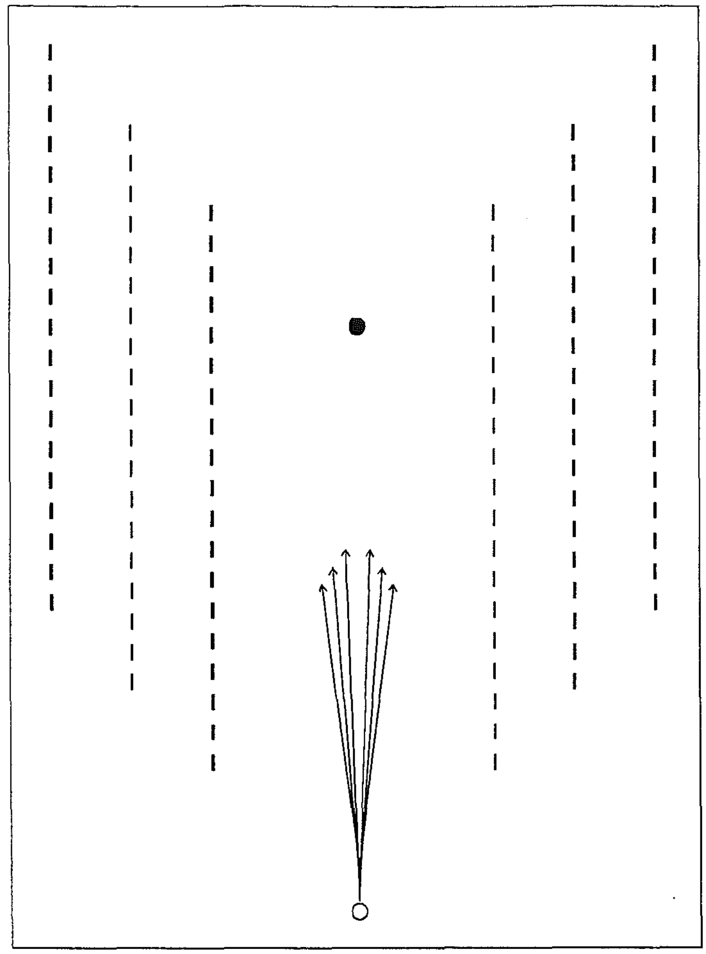

Figure 4. Schematic representation of the differential motion parallax environment viewed from above. The open circle represents the position of the simulated egolocus at trial onset. The filled circle represents the target position. The six arrows represent the six possible initial deviations of the movement direction from the target. Each of the six dashed lines represents a row of objects in the environment.

Random Environment. The Random environment was constructed by adding 108 objects positioned randomly with constraints to the Target-only environment. The resulting simulation was rich in the types of information potentially useful for the detection of heading direction (Figure 5).

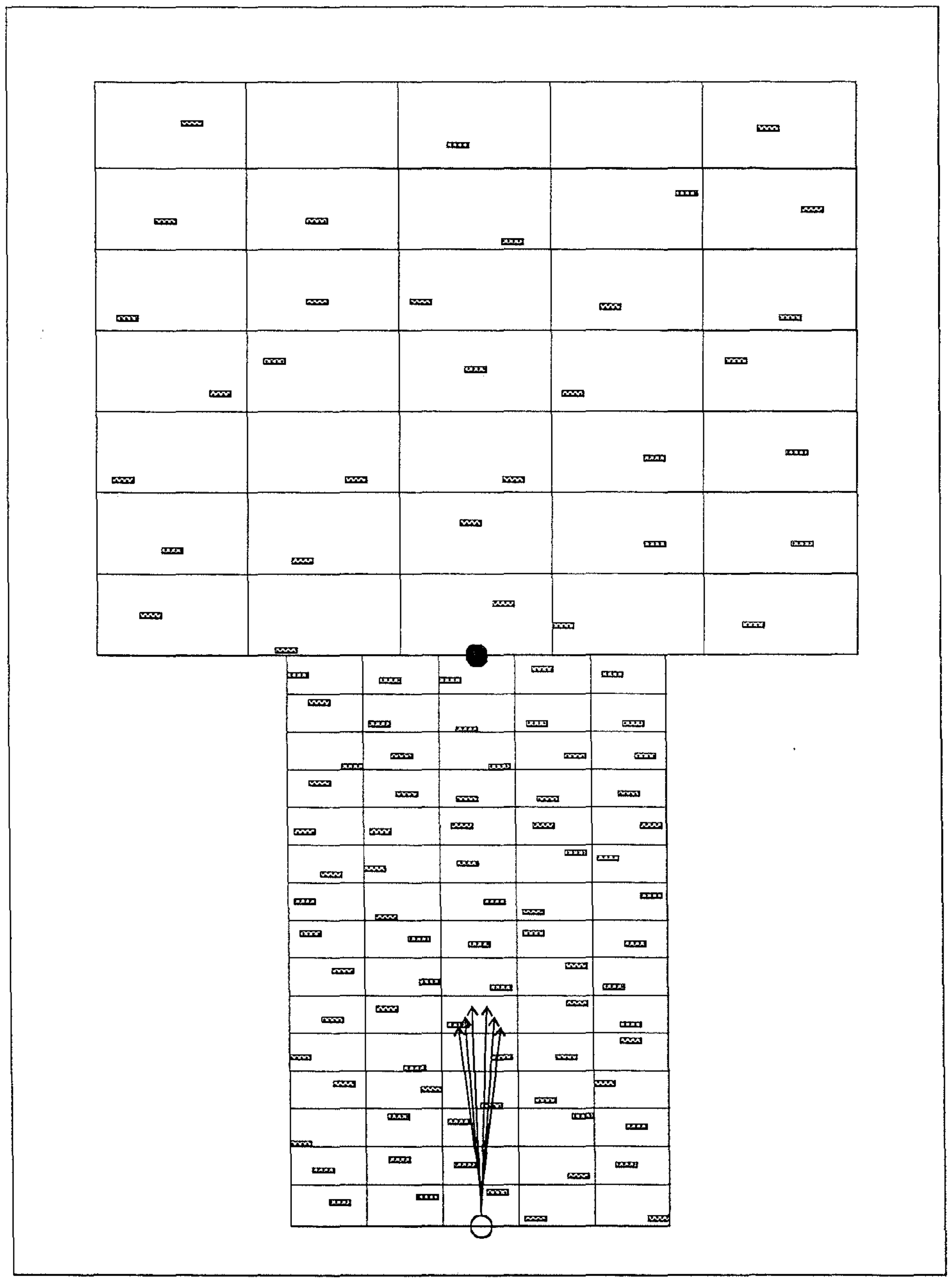

Figure 5. Schematic representation of the Random environment viewed from above. The open circle represents the position of the simulated egolocus at trial onset. The filled circle represents the target. The filled rectangles represent objects other than the target. One object was randomly placed into 108 of the 110 squares (here represented as open rectangles). The squares were not visible during simulation.

(3) Design

There were three simulated environments (Target-only, DMP, or Random). Initial lateral deviation of the target from the center of the display was either -4, -2, 0, 2, or 4 deg, and initial heading error was either -6, -4, -2, 2, 4, or 6 deg, resulting in 90 distinct events.

(4) Procedure

Participants were instructed to attempt to control the simulation such that they were only to engage in control actions if and when they perceived the simulated egolocus to be moving in a direction that did not lead to the target in a straight line. In that case they were to correct the direction of simulated self-motion such that they were satisfied that straight-line travel would eventually reach the target. All instructions were presented on the simulation display in text form, and participants advanced to subsequent screens by squeezing a trigger on the joystick (see Appendix B for the complete set of instructions).

Before the start of the experimental trials, participants were presented with five demonstration trials and six practice trials. The initial conditions of all five demonstration trials illustrated straight-line travel towards the target. To facilitate questioning, all demonstration trials were conducted without use of the viewing hood. In the first demonstration trial, the focus of radial outflow (FRO) coincided with the center of the display. In the second, it appeared at 10 deg to the left of the center of the display. In the third, a repeat of the second, participants were encouraged to manipulate the control to get a feel for how it reacted. The third and fourth demonstration trials were identical to the second and third, except that the FRO appeared at 10 deg to the right of the center of the display.

At this point the viewing hood was fitted, and participants were instructed to begin with the six practice trials during which each environment was encountered twice. Before beginning the 90 experimental trials participants were again encouraged to ask questions, and were then told to proceed at their own pace.

(a) <u>Trial Presentation</u>. Order of trial presentation was counterbalanced and randomised with constraints, using a procedure described in detail in Appendix C.

II. RESULTS

(1) Preparation of Data and Exploratory Analysis

The output of the joystick and the resulting position in the virtual environment were recorded at 60 Hz. For each time sample, the main dependent variable, deviation of the instantaneous direction of simulated self-motion from the target or heading error (θ) was calculated. As the target was passed, this measure increased sharply up to $|180|$ deg. Since this increase is not a function of performance, the data sets were terminated 60 samples before θ reached a value of $|90|$ deg.

Next, the data sets were checked for 'spikes', i.e., recordings of episodes lasting less than three samples. These recordings do not reflect actual control episodes, but are the result of noise exceeding the deadband. In total, 87 such spikes were found and subsequently all such values were replaced with 0.

Depending on the initial conditions, $|\theta|$ could take the value of 2, 4, or 6 deg at the onset of a trial. Until appropriate control was exerted, these initial deviations increased at differing rates. To compensate for this differential increase, a segment was also excluded from the beginning of each trial. The overall mean reaction time measured from the onset of a trial to the onset of the first correct control episode, i.e., a control action that decreased $|\theta|$ or slowed its increase, was found to be 1.7 s (sd = 1.2 s). The corresponding figures for the Target-only, DMP, and Random environments were 2.05 s (sd = 1.36 s), 1.63 s (sd = 1.17 s), and 1.44 s (sd = 0.95 s), respectively, and for absolute initial heading error ($|\theta_i|$) of 2, 4, and 6 deg they were 2.4 s (sd = 1.54 s), 1.47 (sd = .9238 s), and 1.25 s (sd = .58 s), respectively. Consequently, unless where explicitly stated otherwise, the first 2 s of each trial were excluded from further analysis. This cut-off point is further justified by an inspection of mean $|\theta|$ for each level of $|\theta_i|$ as a function of time (Figure 6). Two seconds into the trial, the differential effect of the varying levels of $|\theta_i|$, as evidenced by the different initial slopes of the three lines during the first second had been compensated for to some extent. Somewhat surprisingly, the differences in $|\theta_i|$ tended to have a lasting effect on performance. The increase in slope towards the end of the trials indicates the beginning of the exponential increase discussed in the first paragraph of this section.

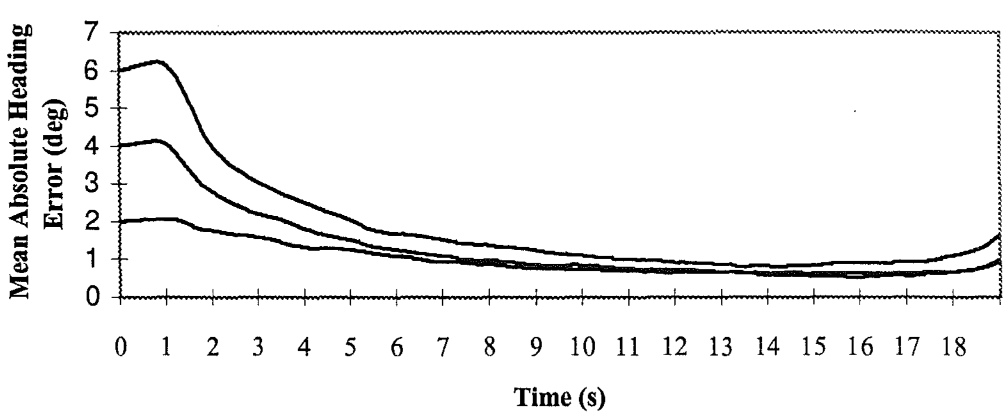

Figure 6. Mean absolute heading error (mean $|\theta|$) as a function of time, grouped by the three levels of absolute heading error at trial onset ($|\theta_i|$) (each point on a curve represents the average of 180 observations).

To investigate the possibility that subjects engaged in a considerably higher frequency of control episodes in the beginning and towards the end of a trial (an occurrence that would indicate that the data should be split into two or more distinct segments for further analysis), a cumulative frequency plot of control onsets was constructed. As can be seen in Figure 7, the only period of time during which the curves deviate from their essentially linear relationship is during the first 2 s of a trial. Since, as discussed above, this segment had already been excluded from further analysis, the data were not subdivided any further.

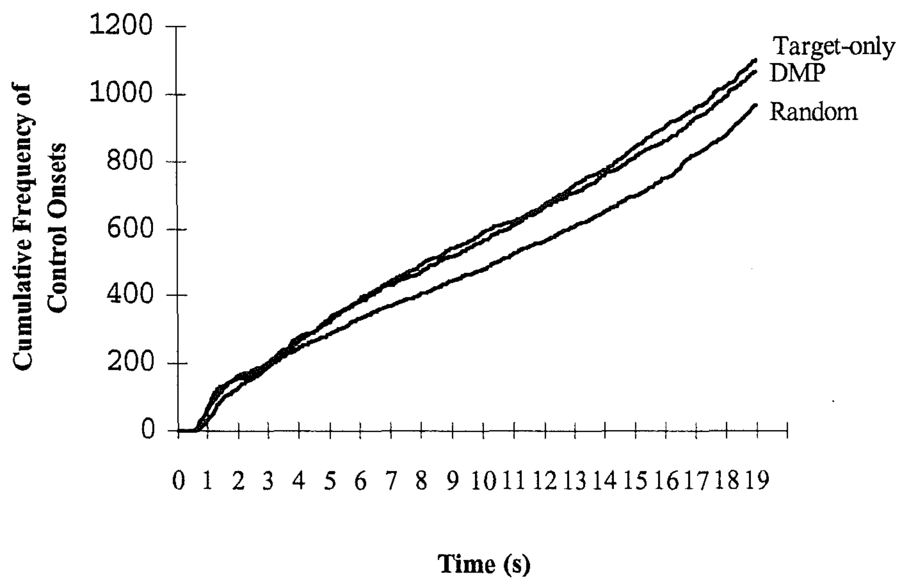

Figure 7. Overall cumulative frequency of control onsets as a function of time and the three experimental environments.

After visual exploration of the data from a number of individual trials the following variables were extracted from the raw data of each trial for further analysis: (a) elapsed time in seconds to onset of first correct control episode, (b) mean $|\theta|$

throughout the entire truncated trial[10], (c) mean $|\theta|$ during zero-control episodes, (d) mean $|\theta|$ and associated standard deviations at both the initiation and the conclusion of control episodes, and (e) number of both correct and incorrect control episodes during a trial[11].

(2) Analysis of the Extracted Variables

To evaluate whether the data could be collapsed over left and right initial heading errors (θ_i) and left and right deviations of FRO from the center of the display ($FRO_{dev\text{-}C}$), a multivariate analysis of variance (MANOVA) of all dependent variables of the form θ_i (left or right) x $FRO_{dev\text{-}C}$ (left or right) was conducted. Both main effects were nonsignificant (Pillais $F(9, 456) = .6736$, ns, and Pillais $F(9, 456) = .4683$, ns, respectively). Consequently, the data were collapsed over left and right cases for the two variables in question. However, the analysis also yielded a significant multivariate interaction effect (Pillais $F(9, 456) = 2.6945$, $p=.005$). The associated univariate results (Appendix D, Table D-1) indicated that this difference was mainly due to differences in variables extracted from θ. The corresponding distribution of means was nearly identical for all variables and indicated that performance was superior when θ_i and $FRO_{dev\text{-}C}$ had opposite signs, i.e., either left and

[10] The companion variable, rate of change of θ, was found to closely mirror the values of the control vectors. During zero control episodes, θ was typically very small (in the range of -1 to 1 deg). In this case, the rate of change of θ is near enough constant and very close to zero. It was therefore decided to concentrate on the analysis of the control values instead.

[11] Note that three of the extracted variables (number of correct and incorrect control episodes, as well as elapsed time to onset of first control episode) have been extracted from data sets that include the first 2 s of a trial.

right, respectively, or vice versa. As an example, the relevant values for the means of $|\theta|$ at the conclusion of control episodes[12] are represented in Figure 8. This result is indicative of the existence of an aperture bias, i.e., the tendency to perceive the aimpoint to be closer to the center of the display than it actually is. A more detailed analysis of this bias is presented later (see p. 44).

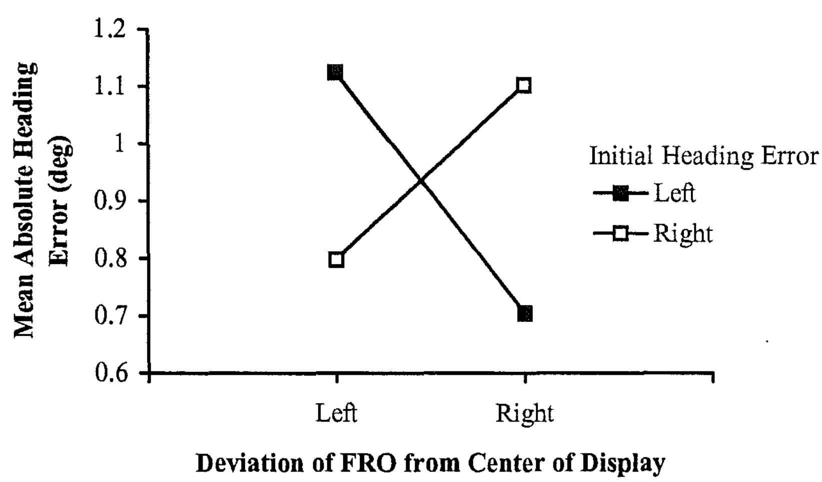

Figure 8. Mean absolute heading error (mean $|\theta|$) at the conclusion of control episodes as a function of deviation of FRO from center of display and initial heading error (θ_i).

In addition to θ_i, the above analysis employs $FRO_{dev\text{-}C}$ to describe the simulation. An alternative to this variable is given by the initial deviation of the target from the center of the display ($T_{dev\text{-}C}$). Again, to evaluate whether the data could be

[12] This measure was chosen since it is the best available indicator of a threshold for the perception of heading.

collapsed over left and right for this variable, it was substituted for $FRO_{dev\text{-}C}$ in the above MANOVA. The main effect for $T_{dev\text{-}C}$ was found to be nonsignificant (Pillais $F(9, 420) = .8459$). Although the multivariate main effect for $FRO_{dev\text{-}T}$ was found to be significant (Pillais $F(9, 420) = 2.1583$, $p<.05$), the fact that none of the relevant univariate tests approached significance (Appendix D, Table D-2) together with the results of the previous analysis was taken to indicate that the data could remain collapsed across left and right of θ_i. Again, a significant interaction effect (Pillais $F(9, 420) = 8.2532$, $p < .001$) suggested the existence of an aperture bias. The pattern of means is adequately represented by Figure 8 (substituting $T_{dev\text{-}C}$ for $FRO_{dev\text{-}C}$). The relevant univariate test results are presented in Appendix D, Table D-3.

The main analysis of the extracted variables was conducted by means of a one-way multivariate analysis of covariance (MANCOVA) of all continuous dependent variables of the form Environment (Target-only, DMP, or Random), with $|T_{dev\text{-}C}|$, $|FRO_{dev\text{-}C}|$, $|\theta_i|$, and trial number as covariates. Analysis of the covariates showed a significant overall multivariate effect (Pillais $F(36, 2112) = 13.8383$, $p<.001$). The univariate tests showed that all dependent variables were strongly affected (Appendix D, Table D-4). The signs of the associated regression coefficients (Table 1) indicate that performance generally improved with practice and deteriorated with increasing $|FRO_{dev\text{-}C}|$ and $|\theta_i|$.

The analysis also yielded a significant effect for Environment (Pillais $F(18, 1052) = 4.5965$, $p<.001$). With the exception of number of incorrect control episodes, all univariate effects proved to be strongly significant (all $p<.001$, for details see Appendix D, Table D-5). The overall pattern of results suggests that performance

Table 1. Regression coefficients for the dependent variables associated with each of the covariates.

	Covariates			
Dependent variables	$\lvert FRO_{dev\text{-}C}\rvert$	$\lvert\theta_i\rvert$	$\lvert T_{dev\text{-}C}\rvert$	Trial
Mean $\lvert\theta\rvert$ (deg)	.106**	.061**	.028	-.006**
Mean $\lvert\theta\rvert$ during zero-ctrl	.102**	.040*	.020	-.006**
Mean $\lvert\theta\rvert$ at ctrl conclusion	.095**	.080**	.037*	-.006**
.....Associated SD	.052**	.081**	.025	-.003**
Mean $\lvert\theta\rvert$ at ctrl initiation	.135**	.080**	.027	-.009**
.....Associated SD	.060**	.088**	.008	-.002*
Reaction time	.127**	-.387**	.077**	-.003
No. of correct ctrls	-.026	.243**	-.087	.006
No. of incorrect ctrls	.034**	-.021	.040*	-.001

Note. $\lvert FRO_{dev\text{-}C}\rvert$ = deviation of FRO from center of display (constant throughout trial), $\lvert\theta_i\rvert$ = initial heading error, $\lvert T_{dev\text{-}C}\rvert$ = initial deviation of target from center of display.

* p<.05, ** p<.001

in the Target-only environment was poorer than in the other two environments (Table 2). Although there were no significant differences between the DMP and the Randomenvironments in terms of mean $\lvert\theta\rvert$, the mean elapsed time to the first correct control episode was shorter in the Random environment than in the DMP environment. Finally, the mean number of correct control episodes was significantly less in the Random environment than in the remaining two environments.

Table 2. Differences between means for the three environments (for actual means see Appendix D, Table D-6).

Variable	Target-only - DMP environment	Target-only - Random environment	DMP - Random environment
Mean |θ| (deg)	0.29***	0.30***	0.01
Mean |θ| during zero-ctrl	0.26***	0.25***	-0.01
Mean |θ| at ctrl conclusion	0.26***	0.28***	0.02
.....Associated SD	0.13**	0.19***	0.06
Mean |θ| at ctrl initiation	0.41***	0.41***	0.01
.....Associated SD	0.19***	0.28***	0.10
Reaction time (s)	0.42***	0.61***	0.18*
No. of correct ctrl episodes	0.23	0.77***	0.54**
No. of incorrect ctrl episodes	-0.04	-0.03	0.01

*p<.05, **p<.01, ***p<.001

Finally, since the sample size was relatively small, it is instructive to briefly compare the individual results with the group effects. The results for mean |θ| at the conclusion of control episodes are used as a representative example (Table 3). All subjects showed the group effect for the comparison between the Target-only and the DMP environment, and all but one subject showed the group effect for the comparison between the Target-only and the Random environment. There were no significant group differences for the comparison between the DMP and the Random environment. Since there will always be *some* difference for individual subjects, the classification becomes a little problematic. However, it seems fair to suggest that four of the six subjects showed negligible performance differences.

Table 3. Differences between mean $|\theta|$ at the conclusion of control episodes for the three environments and the six participants.

Subject	Target-only - DMP environment	Target-only - Random environment	DMP - Random environment
1	0.44*	0.41*	-0.04*
2	0.31*	0.32*	0.01*
3	0.13*	0.50*	0.37
4	0.28*	-0.09	-0.37
5	0.16*	0.27*	0.11*
6	0.24*	0.31*	0.07*

* Difference between means is in the same direction as the group effect.

(3) Aperture Bias

If the aimpoint is perceived to be nearer to the center of the display than it actually is (aperture bias) when FRO is to the left of the screen center ($FRO_{dev\text{-}C}$ is negative), mean absolute heading error to the left of the target (mean $|\theta_L|$) should be greater than mean absolute heading error to the right of the target (mean $|\theta_R|$). Conversely, when FRO is to the right of the screen center ($FRO_{dev\text{-}C}$ is positive), mean $|\theta_L|$) should be smaller than mean $|\theta_R|$. This prediction was tested in an ANOVA of mean $|\theta|$ during zero-control of the form $FRO_{dev\text{-}C}$ (-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, or 10 deg) x sign of heading error (negative = left, or positive = right). The analysis yielded a significant interaction effect ($F(10, 1058) = 37.4$, $p<.001$), indicating the presence of an aperture bias (Figure 9).

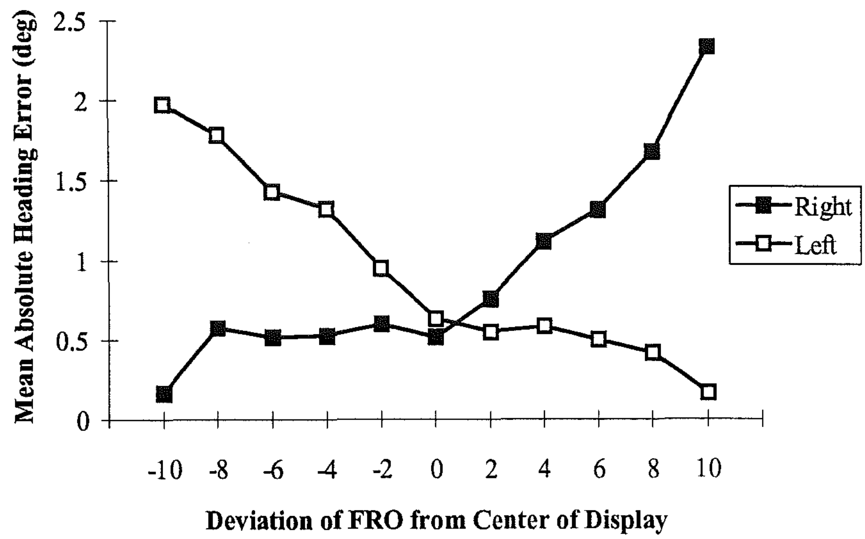

Figure 9. Mean absolute heading error (mean $|\theta|$) as a function of deviation of FRO from center of display (negative sign indicates left, positive sign indicates right) and sign of θ.

More specifically, when $FRO_{dev\text{-}C}$ is negative, heading error to the left of the target is greater than heading error to the right of it. This relationship is reversed for positive $FRO_{dev\text{-}C}$. In addition, this difference increased with increasing $|FRO_{dev\text{-}C}|$ ($F(10, 1058) = 7.25$, $p<.001$). In other words, the aimpoint was perceived to be closer to the center of the display than it actually was. This bias seemed to increase with increasing aimpoint deviation from the center of the display. There were no significant differences between signs of heading error ($F(1, 1058) = .16$, ns), indicating that overall, there was no tendency to perceive the aimpoint more to one side of the actual heading direction than the other.

III. DISCUSSION

(1) Effect of Initial Conditions

This first exploratory study provided a number of interesting results. While it seemed reasonable to expect that differences in absolute initial heading error ($|\theta_i|$) would rapidly dissipate, Figure 6 indicates that they influenced performance throughout the entire trial duration. The results of the analysis of covariance confirm this suggestion, i.e., all measures derived from heading error increased and became more variable with increasing $|\theta_i|$[13]. Perhaps some form of 'perceptual anchoring' took place. In other words, the criterion for a heading error that is 'good enough' may have been influenced by the initial condition for the duration of an event. A small initial heading error may have led to a more conservative criterion for acceptable performance than a large one. Alternatively, due to system constraints it may have taken longer to compensate for initial differences in heading error than the initial 2 s that were eliminated from the trials. In other words, although the participants may have been aware of heading in the wrong direction, the kinematics of the system (e.g., level of gain) may have been such that the desired heading direction was difficult to achieve.

The corresponding results for initial deviation of the target from the center of the display ($|T_{dev\text{-}C}|$) are more difficult to interpret. Overall, as expected it seems that performance varied relatively little with this variable. Nevertheless, as indicated by a slight increase in three of the dependent variables (mean heading error at the

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[13] The significant increase in the mean number of correct control episodes is taken to be indicative of the fact that a larger $|\theta_i|$ requires more control input to achieve a satisfactory result.

conclusion of control episodes, reaction time, and number of incorrect control episodes) there is a suggestion that the task was performed less successfully with large values of $|T_{dev\text{-}C}|$.

(2) Aperture Bias

The results of the analysis of covariance further indicated that the experimental task became more difficult with increasing absolute deviation of FRO from the center of the display ($|FRO_{dev\text{-}C}|$). This finding replicates results obtained by R. Warren (1976), who showed that pointing performance in response to a random dot display deteriorated by about 0.3 deg for each degree that $|FRO_{dev\text{-}C}|$ increased. The tested range was 0, 15, 30, 45, 60, 75, or 90 deg, and performance was most accurate at either end of this distribution. This fits snugly with the corresponding result of the present experiment of about 0.1 deg, since the tested range was in the lower end of that distribution, i.e., 2, 4, 6, 8, or 10 deg. All measures derived from heading error increased and performance became more variable. In addition, reaction time as well as the mean number of incorrect control episodes increased. Together with the finding that mean absolute heading error toward the center of the display was smaller than that toward the sides of the display, these results may be taken to indicate the existence of an aperture bias, i.e., the tendency to perceive the heading direction to be closer to the center of the display than it actually is. Early research on the perception of heading identified the existence of this bias. In an experiment on aimpoint estimation of a filmed landing approach of an aircraft Gibson (1947, pp. 232-233) reported that people were more likely to accurately judge the aimpoint when it coincided with the center of the frame on the presentation screen. Using a shadow caster to present an expanding flow pattern consisting of dots, Llewellyn (1971) found that people estimated centrally located heading directions with greater accuracy. Similarly, aimpoint estimations at the conclusion of a dome-projected simulation of an approach to a surface consisting of a pattern of dots have been shown to be biased towards the center of the projection surface (Johnston et al., 1973). More recently, Cutting et al. (1992) presented evidence of the existence of the bias for simulated self-motion through an environment consisting of a number of flat disks scattered across an endless ground plane. Although W. H. Warren, Mestre et al. (1991) also found evidence of the bias following simulated circular self-motion across a ground plane consisting of dots (their Experiment 3), they discounted the phenomenon on two grounds. (a) The effect was found to be reversed for circular approaches to a vertical wall, again consisting of dots. Observers reported perceiving the aimpoint between the actual aimpoint and the side of the display. The most plausible hypothesis the authors advanced to explain this 'inside bias' concerns itself with a peculiarity of the flow pattern generated by an approach to a wall. The instantaneous flow pattern generated by such a simulation has a radial structure with a 'pseudofocus of outflow' lying to the inside of a curvilinear path leading to the actual impact point. This point could misinform observers with respect to aimpoint estimation. The authors rejected this explanation since the predicted bias would be considerably larger than the observed one. However, there seems to be no reason to believe that the two biases could not have affected performance at the same time. If this were the case, inside bias induced by the pseudo-FRO could have been reduced to the observed level by the effect of a strong aperture bias[14]. (b) The authors

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[14] Unfortunately, calculation of the predicted inside bias is not possible, since the authors did not report the angular deviations of the pseudo-FRO from the simulated impact points.

reanalized performance data on translational heading originally reported by W. H. Warren, Blackwell and Morris (1989) and found no evidence of the bias. This result notwithstanding, it seems that, on balance, the existing evidence suggests that center-screen or aperture bias does affect heading perception following passive observation of simulated visual self-motion events. Results of the present experiment suggest that heading perception is also subject to this effect during active control of such events.

(3) Target Drift

The results of Experiment 1 indicate that the observed reaction times were shorter than expected. In a passive observation experiment investigating heading perception Cutting et al. (1992) found reaction times between 4 s and 6 s for comparable initial heading errors. It is unlikely that the large differences (up to 4.75 s) are due to the active control component, since until the first control action is made, the simulation is passively observed. However, the two studies differ in an additional important aspect. Cutting et al. (1992) employed a 'dolly-and-pan' simulation technique where the center of the display remained fixed on an object in the simulated environment (simulating eye fixation on an object along the side of a linear movement path). The present simulation, where the angle between heading and viewing direction remained constant throughout a trial, contained target drift, i.e., the apparent movement of the target across the display as a potentially functional type of information. To initiate a control episode in the correct direction, a person sensitive to it would simply have to push the joystick in the opposite direction of the target drift (Llewellyn, 1971). Consequently, it is possible that target drift in itself is easier to detect (with reaction times being faster) than information contained in the type of simulation employed by Cutting et al. (1992). The results from Experiment 2 will allow an evaluation of this possibility.

Even though the Environment main effect indicated that performance in the Target-only environment was poorer than in the remaining two environments, performance was still very good. Cutting et al. (1992) estimated that the required wayfinding accuracy for similar flow velocities is around 3.6 deg[15]. The relevant mean heading error at the conclusion of control episodes (the best available indicator of a threshold for the perception of heading) of 1.2 deg is still well within that range. This also applies to the observed mean heading error at the initiation of control episodes (a measure that might be more suitable for comparisons to passive observation studies) of around 1.8 deg. It is suggested that in the Target-only environment, where only the target was present, target drift was the only functional information for the perception of heading. In other words, to perform the experimental task, a person sensitive to target drift would simply have to correct the simulated path of self-motion such that the target remained stationary with respect to the sides of the display during subsequent zero-control episodes. Of course it is also possible that people were sensitive to (a) the rate of expansion (strategy: maximize this variable to remain on correct heading) (Regan & Beverley, 1982), and/or (b) the information contained in the apparent rotation of the target as it is passed (strategy: null the rotation). To investigate whether target drift or one of the other two variables provides functional information for the perception and control of heading, a different simulation technique was utilized in Experiment 2. Whereas in Experiment 1

[15] Note that according to earlier estimates, an accuracy of around 1 deg is required (Cutting, 1986, p. 152; Cutting, 1991).

the angle between the instantaneous direction of simulated self-motion and the center of the display was kept constant throughout a trial (simulating a person moving forward with the head turned at a constant angle), in Experiment 2 this 'viewing direction' was fixed on the target. In other words, since the simulated viewing direction is centered on the target, target drift is effectively eliminated while the two variables associated with target expansion remain unaffected. However, the difference in rate of expansion between a 'dead on' heading and one that is one or two degrees off is so minimal that it is unlikely to convey any useful information. In addition, the nature of the target employed in the present investigation is such that the optical pattern of the target undergoes only very slight changes with apparent rotation throughout the simulation., i.e., the appearance of the target is such that it looks virtually the same independent of the direction from which it is approached. Consequently, neither rate of expansion nor rotational information were expected to support goal directed locomotion in the Target-only environment.

(4) Differential Motion Parallax (DMP)

As outlined in the introduction, Cutting (1986) proposed that information contained in the retinal velocities of near and far objects with respect to a fixated object in the middle distance is used for the guidance of self-motion. I have proposed an alternative formulation that describes this transformation with reference to the *optic* array. Briefly, DMP is the potential information about the direction of self-motion contained in the differential velocity of discontinuities produced by objects at different distances from the moving point of observation. In order for DMP to occur, there must be at least three objects along a plane of sight[16]. If an observer *attends to* the relative velocities of the three discontinuities, DMP information is acquired (for a more detailed definition see pp. 13-17).

The DMP environment was designed to provide differential motion parallax in the peripheral field of view. This was achieved by keeping a horizontal optical angle of 8 deg around the simulated target free of other objects[17]. In contrast, objects in the Random environment were placed to create a reasonable approximation of a cluttered environment. As such, it also contained DMP information (for details, see Appendix A).

The fact that performance was worse in the Target-only than in the DMP environment suggests the possibility that differential motion parallax information is a type of information for the perception of heading that provides functionality over and above that provided by target drift. Second, the lack of a performance difference between the DMP and Random environments indicates that heading perception derived from target drift and DMP might be as accurate as that provided by a reasonably cluttered environment. However, close scrutiny of the utilized environments led to a number of observations and unanswered questions that the subsequent experiment was designed to answer. (a) As pointed out above, the Target-only environment contains at least three types of information for the perception and control of heading (target drift, rate of target expansion, and target rotation). Experiment 2 was designed to provide indications of the relative

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[16] For a definition of the term 'plane of sight' see p. 31.

[17] Viewed from the initial point of observation, no object was placed within 4 deg on either side of the target. This value could increase to 16 deg by the end of a trial.

importance of these types of information. (b) In the DMP environment these three information types were present as well as the intended DMP information[18]. Experiment 2 will allow an evaluation of performance in an environment containing DMP when target drift is absent. (c) A comparison between Experiment 1 (target drift present) and Experiment 2 (target drift absent) will allow an evaluation of the functionality that target drift provides over and above DMP information.

(5) Global Optical Flow Velocity (GOFV)

Since the new simulation technique allowed for a smaller experimental design (initial deviation of the target from the center of the display was obsolete since the target remained in the center of the display), it became possible to also vary the level of global optical flow velocity (GOFV). At higher speeds time-to-contact with a given object decreases, and the risk of collision with an object as well as the gravity of the consequence of such a collision increases. It would make eminent evolutionary sense to be more sensitive to heading information at higher levels of GOFV. Findings from previous studies seem on balance to be nonequivocal. For example, W. H. Warren et al. (1988) simulated linear self-motion across an endless plane consisting of dots and found small performance improvements over the investigated range of GOFV (0.6, 1.2, and 2.4 h/s), with the difference between the highest and the lowest level of GOFV being significant. Using the same simulation, W. H. Warren et al. (1989) also reported performance improvements with increasing GOFV. This improvement was found to be largest for people in an age range similar to the

[18] The relative importance of additional sources of information that are also contained in this environment is addressed in Experiment 3.

participants of the present investigation. In addition, W. H. Warren, Mestre et al. (1991) simulated circular self-motion across an endless dot plane and found improved performance at higher levels of GOFV (investigated levels of GOFV: 0.6, 1.2, 1.8, and 2.4 h/s). In contrast, the reverse effect was observed when a circular approach to a wall consisting of dots was simulated. Similarly, Turano and Wang (1994) found that the ability to distinguish between a curved and a straight path (simulating motion through a cloud of dots) decreased with increasing GOFV. Similarly, evidence from an active control study indicated that sensitivity to loss in altitude during self-motion decreased with increasing GOFV (Wolpert & Owen, 1987). It will be interesting to see what effect different levels of GOFV in an active control paradigm during self-motion at constant altitude might have. Since existing evidence did not allow a prediction either way, it seemed best to go with the intuitive argument presented at the outset of this section and to predict improving accuracy of heading perception during simulated self-motion with increasing GOFV. Given the richness of information gained from active control trials it should also be possible to test a further prediction: People have not evolved to cope with optic flows that are significantly faster than those experienced when we move around on foot. Although it makes consequentialist sense to become more sensitive to heading information at greater flow velocities, this increased sensitivity should be subject to a floor effect within the speed range of human bipedal locomotion. In other words, it is expected that sensitivity to heading error optimizes within that range.

CHAPTER III

EXPERIMENT 2

I. METHOD

Only differences to Experiment 1 are reported.

(1) Participants

Five of the participants of Experiment 1 also took part in Experiment 2. One more male subject (21 years of age) was recruited.

(2) Apparatus

(a) Kinematics. Global optical flow velocity was set at 0.6, 1.2, or 2.4 h/s. Consequently, the target was passed after approximately 40, 20, or 10 s, respectively. These values were chosen to cover the range of GOFV normally experienced by humans during bipedal locomotion. At an eyeheight of 1.7 m this equates to a slow walk, a jog, and a fast run, respectively. The viewing direction always remained centered on the simulated target, simulating 'turning of the head' as the target was passed.

(3) Design

There were three levels of GOFV (0.6 , 1.2 , or 2.4 h/s) and three simulated environments (Target-only, DMP, or Random). Initial heading error (θ_i) was either -6, -4, -2, 2, 4, or 6 deg, resulting in 54 events. The consequent experimental design was a 3 by 3 factorial crossing of GOFV and Environment with $|\theta_i|$ and trial number as covariates.

(4) Procedure

The content of the instructions was kept as close as possible to that used in Experiment 1. Before the start of the 54 experimental trials participants were presented with four demonstration trials and six practice trials. A complete set of instructions containing a description of the demonstration and practice trials is presented in Appendix E.

(a) Trial Presentation. Order of trial presentation was counterbalanced and randomised with constraints, using a procedure described in detail in Appendix F.

II. RESULTS

(1) Preparation of Data and Exploratory Analysis

In total, 33 spikes were found in the data and subsequently replaced with 0 entries. As Figure 10 illustrates, the experimental task in the Target-only environment proved to be so difficult as to make its successful completion impossible. The same pattern of results was observed independent of the level of global optical flow velocity (GOFV). As evidenced by the irregularities in the increase of mean $|\theta|$ for the Target-only environment, there is an indication that some functionality is contained in its two types of information for the perception of heading (rate of optical expansion of target and target rotation). The frequency and duration of these irregularities seem to decrease with increasing GOFV. However, performance is obviously so poor as to make goal oriented self-motion impossible (e.g., mean heading error at the conclusion of control episodes = 17.19 deg). This somewhat extreme result obviates the need for statistical analysis concerning the comparison between the Target-only environment and the remaining two environments. To illustrate further, the descriptive statistics concerning one of the dependent variables are presented in Table 4. The mean reaction time for events using the Target-only environment appears to be approximately four to seven times longer than in the remaining events. More importantly, in 66 of the 108 events with the Target-only environment no control actions were initiated. Apparently, the participants were not able to perceive their heading with respect to the target. The fact that in 42 of the events some control activity was engaged in, can be taken as indicative of attempts to make information available by active exploration.

Consequently, the remaining analysis concerns itself exclusively with data from the DMP and Random environments.

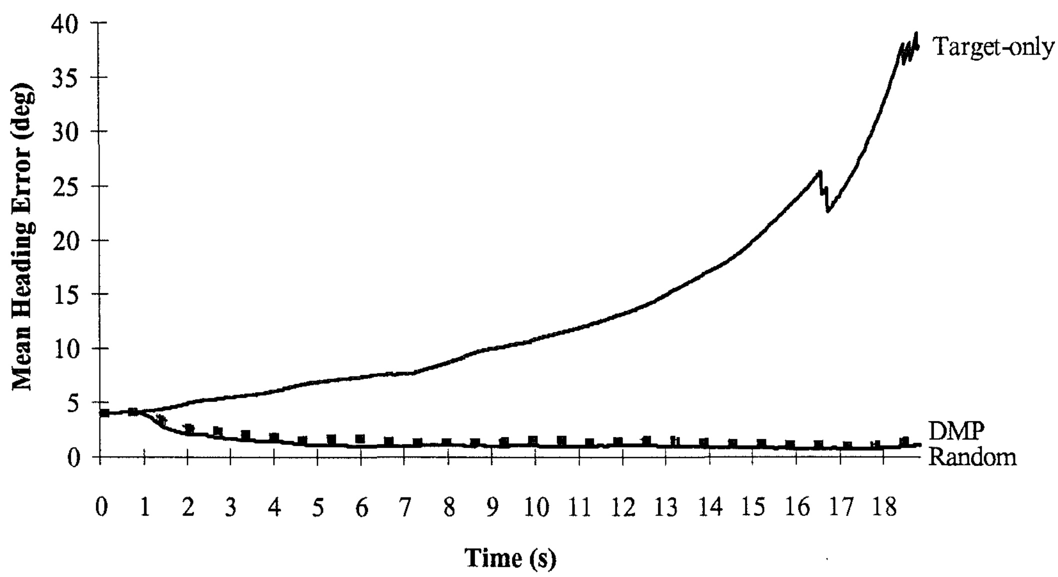

Figure 10. Mean absolute heading error (mean |θ|) as a function of time and the three environments. Global optical flow velocity = 1.2 h/s.

As in Experiment 1, the data sets were terminated 60 samples before the deviation of the instantaneous direction of simulated self-motion from the target [heading error (θ)] reached a value of |90| deg. Similarly, to compensate for the differential increase of |θ| until an appropriate control action was initiated, the first 2 s of each trial were again excluded from further analysis. Although the slightly higher reaction times (Table 4) would have indicated a longer exclusion interval, it was decided to keep the same cut-off point to facilitate comparison[19]. In addition, Figure 11 indicates that the differences related to the differences in absolute initial heading error ($|\theta_i|$) had to a large extent already been compensated for by the end of the 2 s. Unlike in Experiment 1, the effect of $|\theta_i|$ did not seem to have a lasting effect on performance. However, the curves in Figure 11 consist of a smaller number of observations than the comparable means of Experiment 1 (cf. Figure 6). This explains the relative lack of smoothness of the curves, and indicates that any existing trends would not be as easily discernible.

Table 4. Mean times to onset of first correct control episode (in s) for Experiment 2.

	Mean	Std. Dev.	Cases	Mean Exp. 1	Cutting et al. (1992)
Target-only environment	12.18	10.71	42	2.05	
DMP environment	3.67	4.95	108	1.63	
Random environment	1.73	2.00	108	1.44	
For Entire Population	2.70	3.89	216	1.54	
(DMP & Random environments)					
$\|\theta_i\| = 2$ deg	3.68	4.40	72	2.4	4.8
$\|\theta_i\| = 4$ deg	2.74	4.34	72	1.47	3.8
$\|\theta_i\| = 6$ deg	1.68	2.37	72	1.25	3.2
GOFV = 0.6 h/s	4.67	5.87	72		
GOFV = 1.2 h/s	2.18	2.11	72		
GOFV = 2.4 h/s	1.25	0.72	72		

Note. θ_i = initial heading error.

[19] For comparison purposes, the reaction times observed in Experiment 1, as well as those obtained by Cutting et al. (1992, p. 59, Figure 14, Experiment 2) have been included in the table.

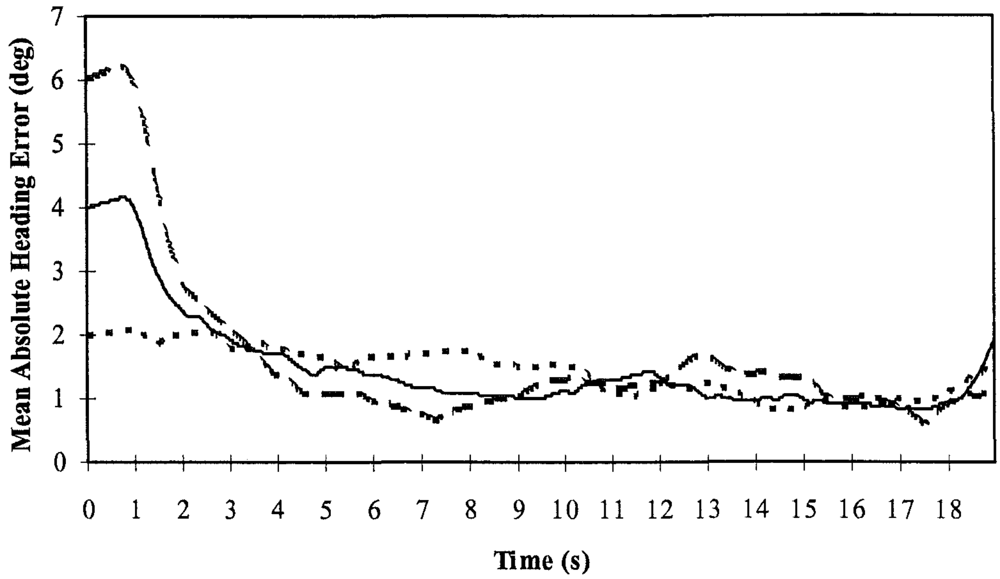

Figure 11. Mean absolute heading error (mean $|\theta|$) as a function of time and the three levels of absolute heading error at trial onset ($|\theta_i|$). Global optical flow velocity = 1.2 h/s (each point on a curve represents the average of 72 observations).

(2) <u>Analysis of the extracted variables</u>

The same variables as in Experiment 1 were extracted from the raw data. To evaluate whether the data could be collapsed over left and right θ_i, a one-way MANOVA of all dependent variables of the form θ_i (left or right) was conducted. The main effect was nonsignificant (Pillais $F(9, 206) = .58864$, ns). Consequently, the data were collapsed over left and right θ_i for the subsequent analyses.

The main analysis of the extracted variables was conducted by means of a MANCOVA of all continuous dependent variables of the form Environment (DMP or

Random) x GOFV (0.6 , 1.2 , or 2.4 h/s) with $|\theta_i|$ and trial number as covariates[20]. Analysis of the covariates yielded a significant overall multivariate effect (Pillais $F(18, 402) = 2.5631$, $p<.001$). The univariate tests show that this effect was mainly due to reaction time ($F(2, 208) = 145.0601$, $p<.05$). The relevant regression coefficient associated with $|\theta_i|$ ($\beta_{(est)} = -.4983$ s) indicates that reaction time decreased with increasing initial heading error. No further significant differences were detected.

The analysis also yielded a significant effect for Environment (Pillais $F(9, 200) = 4.5965$, $p<.01$). With the exception of four of the dependent variables (standard deviation of mean $|\theta|$ both at the initiation and conclusion of control episodes, and number of both correct and incorrect control episodes) all univariate tests proved to be significant ($p<.001$, for details see Appendix G, Table G-1). The overall pattern of results suggests that performance in the DMP environment was poorer than in the Random environment (Table 5). In addition, a look at Table 6 shows that the individual results of all subjects mirrored the group effect.

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[20] It would have also been appropriate to introduce GOFV into the analysis as a covariate. However, four of the dependent measures (mean $|\theta|$ at the conclusion of control episodes, the associated standard deviation, the standard deviation associated with mean $|\theta|$ at the initiation of control episodes, and reaction time) were found to deviate significantly from linearity across the three levels of GOFV (all $p<.05$). The means (Table 7) indicated that performance on those variables improved from 0.6 to 1.2 h/s but not from 1.2 to 2.4 h/s. Although a $\log_{10}$ transformation of GOFV was successful in reducing deviations from linearity, the more conservative factorial approach was used instead.

Table 5. Means and differences between means for the two environments.

Variable	DMP environment	Random environment	DMP - Random environment
Mean $\|\theta\|$	1.71	1.07	0.64*
Mean $\|\theta\|$ during zero-ctrl	1.68	1.03	0.65*
Mean $\|\theta\|$ at ctrl conclusion	1.47	0.93	0.54*
Mean $\|\theta\|$ at ctrl initiation	2.52	1.61	0.91*
Reaction time (s)	3.67	1.73	1.94*

* $p<.001$.

Table 6. Differences between mean $|\theta|$ at the conclusion of control episodes for the two environments and for the six participants.

Subject	DMP environment	Random environment	DMP - Random environment
1	1.13	0.91	0.22*
2	2.81	1.26	1.55*
3	0.80	0.69	0.11*
4	1.42	1.29	0.13*
6	0.53	0.44	0.09*
7	2.10	0.97	1.13*

Note. To facilitate comparisons between experiments, each subject was assigned one identification number for the duration of the investigation.

* Difference between means is in the same direction as the group effect.

Second, a significant main effect for GOFV was detected (Pillais $F(18, 402) = 1.5521$, $p<.001$). All dependent variables were strongly affected (for details see Appendix G, Table G-2). The overall pattern of results shows that performance improved with increasing GOFV (Table 7), especially so from 0.6 to 1.2 h/s. The lessening of this effect for comparisons between GOFV of 1.2 and 2.4 h/s is suggestive of the predicted floor effect. However, the difference in the mean number of both correct and incorrect control episodes is more likely to be due to the systematic variation of trial duration associated with differing levels of GOFV.

Table 7. Means and differences between means for the three levels of global optical flow velocity for Experiment 2.

	Global optical flow velocity (h/s)					
Variable	0.6	1.2	2.4	0.6 - 1.2	0.6 - 2.4	1.2 - 2.4
Mean |θ|	1.67	1.28	1.21	0.39***	0.47***	0.08
Mean |θ| during zero-ctrl	1.65	1.28	1.14	0.37***	0.51***	0.14
Mean |θ| at ctrl conclusion	1.57	1.00	1.01	0.57***	0.56***	-0.01
.....Associated SD	1.19	0.61	0.60	0.58***	0.59***	0.01
Mean |θ| at ctrl initiation	2.51	1.94	1.75	0.56***	0.76***	0.20
.....Associated SD	1.41	0.80	0.60	0.61***	0.81***	0.20*
Reaction time (s)	4.68	2.18	1.25	2.49***	3.43***	0.94***
No. of correct ctrls	5.71	4.58	3.53	1.13***	2.18***	1.06***
No. of incorrect ctrls	0.39	0.18	0.07	0.21*	0.32***	0.11

* p<.05, ** p<.01, *** p<.001

The individual results for mean |θ| at the conclusion of control episodes are represented in Table 8. They indicate that three of the subjects (2, 3, and 7) mirror the group effects. The performance of a further two subjects (1 and 4) optimizes at a GOFV of 1.2 h/s. Finally, the performance of subject 6 remained relatively constant. However, the absolute performance level of that participant was extremely good. It is likely that a floor effect has occurred.

Table 8. Mean |θ| at the conclusion of control episodes and differences between means for the three levels of global optical flow velocity and for the six subjects of Experiment 2.

	Global optical flow velocity (h/s)					
Subject	0.6	1.2	2.4	0.6 - 1.2	0.6 - 2.4	1.2 - 2.4
1	1.15	0.90	1.02	0.25*	0.13*	-0.12
2	2.86	1.63	1.62	1.23*	1.24*	0.01
3	1.13	0.55	0.56	0.58*	0.57*	-0.01
4	1.60	0.90	1.56	0.70*	0.04	-0.66
6	0.53	0.52	0.40	0.01	0.13*	0.12
7	2.18	1.50	0.92	0.68*	1.26*	0.58

* Difference between means is in the same direction as the group effect.

Finally, a marginally significant Environment by GOFV interaction effect was detected (Pillais $F(18, 402) = 1.5521$, $p<.07$). The univariate tests showed that this was mainly due to reaction time ($F(2, 208) = 200.7241$, $p<.001$). Reaction times were much larger in the DMP environment than in the Random environment at GOFV = 0.6 h/s, but this difference dissipated with increasing GOFV (Figure 12)[21]. No further significant differences were detected.

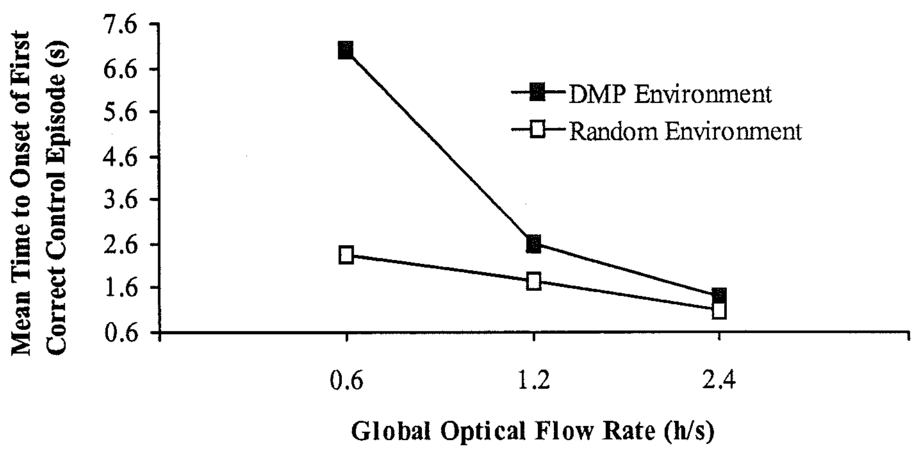

Figure 12. Mean time to onset of first correct control episode as a function of global optical flow velocity and environment.

(3) Comparisons between Experiments 1 and 2

In Experiment 1 a simulation technique was employed where the viewing direction, i.e., the center of the display, was at a constant angle to the simulated instantaneous path of self-motion. In contrast, the viewing direction in Experiment 2 was always centered on the target. A MANCOVA of the form Environment (DMP

[21] A Bonferroni test of multiple comparisons detected significant differences between the mean of the DMP environment (GOFV = 0.6 h/s) and all other means (df = 208, p<.001), and between the mean of DMP environment (GOFV = 1.2 h/s) and that of the Random environment (GOFV = 2.4 h/s).

or Random) x Experiment (1 or 2) of all dependent variables with $|\theta_i|$ and trial as covariates was conducted to investigate the observed differences. A significant within-cells regression effect was detected (Pillais $F(18, 1126) = 9.8214$, $p<.001$). The univariate tests showed that all dependent variables with the exception of the number of incorrect control episodes were strongly affected (all $p<.01$, for details see Appendix G, Table G-3). The relevant regression coefficients associated with each of the dependent variables are presented in Table 9. The overall pattern suggests that increasing initial heading error led to poorer performance (the obvious exception is reaction time which decreases with increasing initial heading error). However, as the previous analyses have shown, this effect was mainly confined to Experiment 1. Similar comments can be made with respect to the practice effect.

The analysis also detected a significant Environment by Experiment interaction (Pillais $F(9, 562) = 4.0354$, $p<.001$). The corresponding univariate tests show that all dependent variables (with the exception of the standard deviations of mean $|\theta|$ at both the initiation and the conclusion of the control episodes, and number of incorrect control episodes) were affected (Appendix G, Table G-4). A nearly identical pattern of effects was detected for the Experiment main effect (Pillais $F(9, 562)$ 5.9751, $p<.001$). While the Environment main effect was also significant (Pillais $F(9, 562) =4.0889$, $p<.001$) the associated univariate results only differed with respect to the standard deviations of mean $|\theta|$ at both the initiation and the conclusion of the control episodes ($F(1, 570) = 281.5557$, $p<.05$, and $F(1, 570) = 248.6026$, $p<.05$, respectively), and the number of correct control episodes ($F(1, 570) = 2341.4317$, ns).

Table 9. Regression coefficients for the covariates associated with each of the dependent variables.

	Covariates	
Dependent variables	$\|\theta_i\|$	Trial
Mean $\|\theta\|$	.112***	-.005**
Mean $\|\theta\|$ during zero-control	.093***	-.005**
Mean $\|\theta\|$ at control conclusion	.108***	-.005**
.....Associated SD	.082***	-.003*
Mean $\|\theta\|$ at control initiation	.140***	-.007**
.....Associated SD	.099***	-.002
Reaction time	-.338***	-.001
No. of correct control epsiodes	.159**	.003
No. of incorrect control episodes	-.011	-.001

Note. θ_i = initial heading error.

* p<.05, **p<.01, *** p<.001.

Interpretation of these effects is best made with reference to the interaction. The most frequent pattern of results is presented in Figure 13. It illustrates that, while performance in the DMP environment was better in Experiment 1 than in Experiment 2, it remained at least constant for the Random environment. A Bonferroni test of multiple comparisons confirms this conclusion (df = 144, p<.001). The exception to this rule is provided by the corresponding figures for the number of correct control episodes (Figure 14). In this case, while this variable again remained constant over Experiment 1 and 2 in the Random environment, it decreased in the DMP environment[22]. Together these two comparisons are taken to indicate that deteriorating performance is associated with a decrease in the number of control episodes in general, and of correct control episodes in particular.

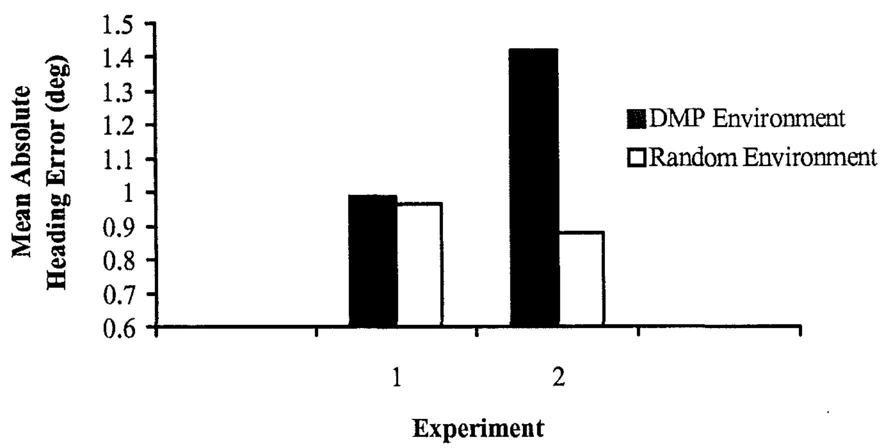

Figure 13. Mean absolute heading error (mean |θ|) at the conclusion of control episodes as a function of experiment number and environment (Experiment 1: Simulated constant angle between center of display and instantaneous direction of self-motion, Experiment 2: Simulated viewing direction fixed on target).

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[22] A Bonferroni test of multiple comparisons detected the following significant differences: (a) The DMP environment (Experiment 1) had a higher value than all other conditions ($p<.05$), and (b) the Random environment (Experiment 1) had a higher value than the DMP environment (Experiment 2) ($p<.05$).

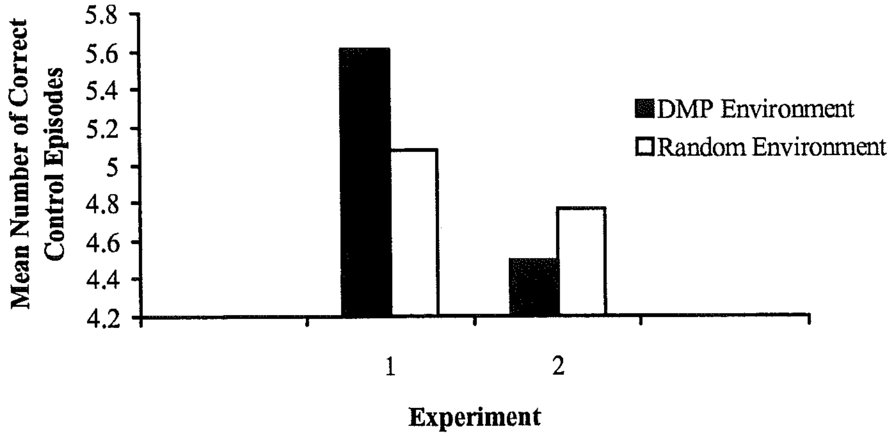

Figure 14. Mean number of correct control episodes during a trial (Experiment 1: Simulated constant angle between center of display and instantaneous direction of self-motion, Experiment 2: Simulated viewing direction fixed on target).

III. DISCUSSION

(1) Rate of Target Expansion and Target Rotation

The most pronounced result of the second experiment was that participants found it impossible to successfully complete the experimental task when only the target was present in the simulated environment. There were two distinct types of information for the perception of heading. (a) The rate at which the target expands is maximized when it is approached in a straight line. This type of information does not depend on the form of the target, i.e., it is expected to be equally ineffective for 'natural' objects (trees, houses, etc.) as for the target in the present study. According to Owen's (1990) classification of the functionality of information contained in the transforming optic array the present study has shown that rate of expansion is a 'noninformative' optical variable for the perception of heading. In other words, although it does carry information that could make goal oriented self-motion possible, active human observers are simply 'unattuned', i.e., not sensitive to it. (b) The optical rotation of a target carries information about the direction of self-motion. Consequently, as long as target rotation is compensated for, the target is approached in a straight line. Unlike the first type of information, the functionality of target rotation is expected to vary with the type of object. For example, a perfectly round, textureless pole, or a flat disk lying on the ground (e.g., Cutting et al., 1992) provides no information of this type. On the other hand, an irregular, texture-rich object (e.g., a tree) might be expected to provide enough of this type of information to make goal-directed approach possible. Since in the present simulation a target was used that was designed to approximate a round pole, the amount of this type of information present in the simulated array was minimal to the extent that it was demonstrably nonfunctional.

(2) Effect of Initial Conditions

In contrast to Experiment 1, initial absolute heading error did not seem to have a lasting effect on performance. The only measure that was found to vary with $|\theta_i|$ was reaction time. This indicates that greater initial heading errors were easier to detect, or that there was a speed-accuracy tradeoff (cf. Owen, 1990). Consequently, the question of why $|\theta_i|$ exerted a lasting influence in Experiment 1, but not in Experiment 2 needs to be addressed. Perceptual anchoring was used as a tentative explanation for the finding in Experiment 1. However, if this had occurred, a similar result should have been observed in Experiment 2. There are two possible explanations for this apparent discrepancy. There is a possibility that perceptual anchoring did not occur during Experiment 1. In this case an alternative explanation might involve control related issues. For example, it could be argued that the chosen level of gain associated with control inputs is more suited to the relatively fine control adjustments necessary to compensate for small $|\theta_i|$, rather than the larger adjustments necessary to compensate for values of $|\theta_i|$ in excess of 4 deg. In addition, it would have to hold that, since different simulation techniques were employed for the two experiments, the chosen level of gain adversely affected performance in Experiment 1, but not in Experiment 2. The alternative explanation holds that perceptual anchoring occurred in both experiments, but that for some reason this effect was not as apparent in Experiment 2. In this case an explanation for the failure to detect such an effect would have to be advanced. In support of this latter scenario is the finding that, although the relevant regression coefficients in Experiment 2 did not reach statistical significance, they were of a magnitude similar to those observed in Experiment 1 and in the predicted direction (mean regression coefficient for all dependent variables derived from $|\theta|$ for both Experiments 1 and 2: $\text{Mean}_{1+2}\ \beta_{est} \cong 0.07$ deg, $\text{sd}(\text{mean}_{1+2}\ \beta_{est}) = 0.02$ deg). In addition, data in Experiment 2 consisted of less than half of the observations in Experiment 1. Consequently, it is possible that there were too few observations in Experiment 2 to allow the detection of a possibly reliable but relatively small effect. Similar comments can be made with respect to the failure to find a significant practice effect in Experiment 2 ($\text{mean}_{1+2}\ \beta_{est} \cong 0.005$ deg, $\text{sd}(\text{mean}_{1+2}\ \beta_{est}) = 0.002$ deg). Experiment 3 will allow an evaluation of these two alternative explanations.

(3) Target Drift

As mentioned in the discussion section of Experiment 1, target drift (the apparent movement of the target across the display) is potentially functional for the perception of heading. If the optical discontinuity produced by a target moves laterally with respect to the frame of the display, the target will be passed on the opposite side (Llewellyn, 1971). Of course this is only the case if the path of self-motion is linear (in the case of the present simulation, during a zero-control episode). A comparison between performance in Experiments 1 and 2 for the Target-only environment indicates that target drift by itself can be utilized to the extent that it makes goal directed self-motion possible. Since the two alternative types of information (rate of expansion and apparent target rotation) were ruled out above, it is suggested that target drift alone accounts for the performance difference of more than 10 deg (mean heading error at the conclusion of control episodes). In fact, the observed value for Experiment 1 of 1.2 deg is well within the required wayfinding accuracy for human self-motion as proposed by Cutting et al. (1992).

Two previous studies investigating target drift have found judgment accuracies that were considerably poorer than performance in the present study would indicate. Llewellyn (1971) obtained accuracies of more than 4 deg, while Johnston et al.'s (1973) results indicated that errors could be as large as 10 deg. Two important methodological differences may explain this apparent discrepancy. (a) Participants in the present study actively controlled the simulated path of self-motion. Two studies in this particular field have indicated that awareness of track vector as well as of orientation during simulated self-motion is better when events are actively controlled (Larish & Anderson, 1991; Laudeman & Johnson, 1994). (b) The task demands were different. Participants in the present study were asked to control the direction of the simulated self-motion with respect to a specific optical discontinuity, while subjects of Llewellyn's and Johnston et al.'s studies were simply asked to indicate their heading after all display motion had seized. It is suggested that, had their subjects been asked to indicate whether they are heading to the left/right of a particular discontinuity during the trial, heading judgments would have been considerably more accurate.

The question that begs answering is whether this target drift is present and functional during *all* types of human self-motion, or confined to simulations and vehicular self-motion, valid only with respect to the frame of a window into an environment. Consider a person locomoting through a sparse environment. An example might be someone skiing down a slope in near white-out conditions. A slalom gate-post becomes visible through the fog. Since skis as well as boots are obscured by deep snow, there is no information to be gained by looking at them. The fact that it is not only possible to avoid colliding with the post, but (for an accomplished skier) no problem to ski past it so close as to actually graze it with the shoulder demonstrates the functionality of this type of information in a realistic situation. The important part of this example is that instead of being sensitive to target drift with respect to the sides of an imposed frame (be it the monitor as a window into a virtual world, or the windscreen of a vehicle as a window into the 'real' world), the person perceives the transformation of the optic array (target drift of the post) with respect to the self. Visually, this transformation can be perceived with respect to the orbit of the eye. Extraretinal information (e.g., occulomotor feedback) may also be of importance (e.g., Royden et al., 1994). On the other hand, the fact that the task is considerably easier when visibility is good is taken to indicate that target drift is by no means the only, or even the most important functional type of information. The fact that there was little performance difference between the cluttered (Random) environments with and without target drift might even be taken to indicate that target drift is used when it is the only information available, and not used at all in an information rich environment. Alternatively, it may be the case that performance is subject to a floor effect at a heading error of around 1 deg. Nevertheless, it is suggested that the present findings support the position that target drift contains sufficient functionality for the perception of heading during events of self-motion where the environment is viewed through a window, as well as at least potentially sufficient functionality during events of human self-motion where that window is not present. Additional supporting evidence comes from the area of robotic vision, where this type of information has apparently found successful application (Huttenlocher, Leventon & Rucklidge, 1994, cited in Cutting, Vishton & Braren, 1995).

The second issue that can now be addressed concerns the reaction time data (Table 4). During Experiment 2, as well as Cutting et al.'s (1992) passive observation study of heading perception, the center of the display remained fixed on an object in the simulated environment. With reference to the previous paragraphs, it is suggested that the increase in reaction time from Experiment 1 to Experiment 2 is due to the lack of target drift in Experiment 2. This substantiates the arguments that (a) target drift is used even in information rich environments and (b) system constraints serve to limit performance in terms of $|\theta|$ at around 1 deg. However, reaction times in Experiment 2 were still considerably faster than in Cutting et al.'s (1992) study. Even when the investigators at this point included a preview period of the first frame of an event of 1 s, the comparable values were still approximately 0.6 s slower. It is possible that methodological differences between the two studies (e.g., frame rate 4.5 - 12.5 Hz as opposed to a refresh rate of 60 Hz in the present study, the use of different simulated objects, different number and placement of objects, etc.) were responsible for the faster reaction times observed in the present investigation.

(4) Global Optical Flow Velocity

The present study provides supporting evidence for previous findings from passive observation studies to the extent that increased global optical flow velocity (GOFV) results in more accurate heading perception (W. H. Warren et al., 1988, 1989, W. H. Warren, Mestre et al., 1991), and extends them to the extent that this also leads to a performance improvement in the active control of heading. In addition, as predicted the present findings indicate that the beneficial effect of increased GOFV was subject to a floor effect in the upper speed range of human bipedal locomotion. On the other hand, these results stand in contrast to evidence from (a) an active control study which found decreased sensitivity to change in altitude during self-motion with increasing GOFV (Owen & Wolpert, 1987), and (b) several passive observation studies which reported decreasing sensitivity to heading information with increasing GOFV (W. H. Warren, Mestre et al., 1991; Turano & Wang, 1994; Hettinger, Owen & R. Warren, 1985; Owen & Freeman, 1987; Wolpert & Owen, 1985). Furthermore, there was a tendency for GOFV to interact with environment type, such that the reaction times in the DMP environment at the lowest GOFV (0.6 h/s) were disproportionally longer than those associated with the remaining conditions. To investigate whether these results have stability, the same levels of GOFV were included in Experiment 3.

(5) Differential Motion Parallax

In Experiment 1 there was no performance difference between the DMP and the Random environment. In contrast, in Experiment 2 performance was better in the Random environment. This result suggests that target drift adds substantial functionality to an event that already carries peripheral DMP information. Second, the Experiment by Environment interaction provided an indication of a possible inverse relationship between the number of correct control episodes and heading error. However, the number of both correct and incorrect control episodes did not differ between the DMP and the Random environment, despite the corresponding improvement in heading error. In addition, results from Experiment 1 (Table 2, p. 43) show that (a) a decrease in the number of correct control episodes from the Target-only to the Random environment was accompanied by a corresponding decrease in heading error and (b) the number of correct control episodes also decreased from the DMP to the Random environment, despite the fact that performance in terms of heading error remained constant. Together these results indicate that the relationship between performance in terms of heading error and number of correct control episodes may not be a stable one.

In addition, the results of Experiment 2 indicate that performance in the DMP environment, where peripheral DMP is present but target drift is absent, is still at a reasonable level (mean $|\theta|$ at the conclusion of control episodes = 1.47 deg). Again, this level of performance is within the required wayfinding accuracy proposed by Cutting et al. (1992) of around 1.86 deg for a forward speed of 4 m/s [assuming an eyeheight of around 1.7 m, this level of speed produces the highest level of GOFV employed in the present Experiment (2.4 h/s)]. Interestingly, performance is almost identical to that in the Target-only environment where only target drift is present.

However, while it has been shown that the Target-only environment (Experiment 1) contains target drift as the only functional type of information for the perception and control of heading, the same cannot be said for the DMP environment (Experiment 2) with respect to DMP. In particular, two additional potentially functional types of information can be identified: The regular arrangement of objects, reminiscent of a runway or road might be useful (Beall & Loomis, 1995b). In addition, so far it has not been shown that simple motion parallax (SMP), i.e., the information contained in the relative optical velocities of only two objects at different distances does not provide sufficient functionality to allow performance at the observed levels. Theoretically, the information contained in such a transformation is sufficient to determine the direction of self-motion (Rieger & Lawton, 1985). To provide answers to these questions, two additional environments were designed.

CHAPTER IV

EXPERIMENT 3

I. METHOD

(1) Participants

All six of the participants of Experiment 1 also took part in Experiment 3. Seven more male subjects between the ages of 22 and 27 years were recruited.

(2) Apparatus

(a) Kinematics. As in the previous two experiments, eyeheight (h) was kept constant, and as in Experiment 2, global optical flow velocity was set at 0.6, 1.2, or 2.4 h/s. The simulation technique was the same as in Experiment 2.

(b) Environments. Based on the conclusive results of the previous two experiments concerning target drift as well as target rotation and rate of target expansion, the environment containing only the target was excluded. Instead, to find answers to the questions raised in the discussion of Experiment 2, two further environments were introduced. Both of the additional environments only differed to those used in the previous two experiments with respect to the number and placement of nontarget objects.

No Motion Parallax (NMP) Environment. This environment was designed to allow the evaluation of task performance in an environment where no motion parallax (NMP) was available throughout the trials. This was achieved by removing four rows of objects from the DMP environment, leaving only the two rows closest to, and on either side of, the target (Figure 15). For a more detailed description of the environment refer to Appendix A.

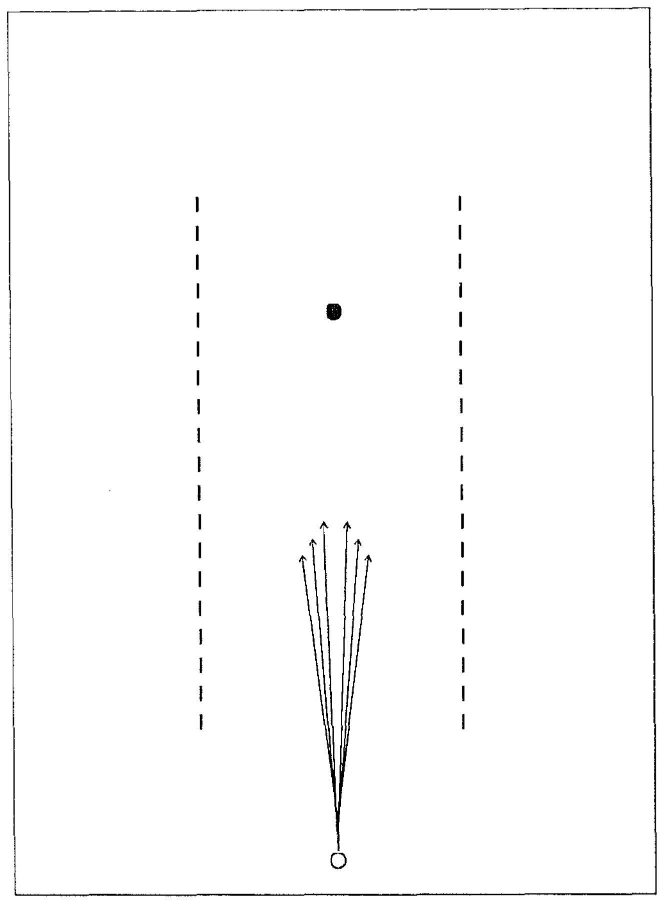

Figure 15. Schematic representation of the NMP environment viewed from above. The open circle represents the position of the simulated egolocus at trial onset. The filled circle represents the target. The six arrows represent the six possible initial deviations of the instantaneous movement direction from the target. Each of the two dashed lines represents a row of objects in the environment.

Simple Motion Parallax (SMP) Environment. To make simple motion parallax (SMP) information available, the middle row was removed from either side of the target of the DMP environment. This left two rows on either side of the target

(Figure 16). The availability of SMP information was the same as that of DMP information in the DMP environment. For a more detailed description of the environment refer to Appendix A.

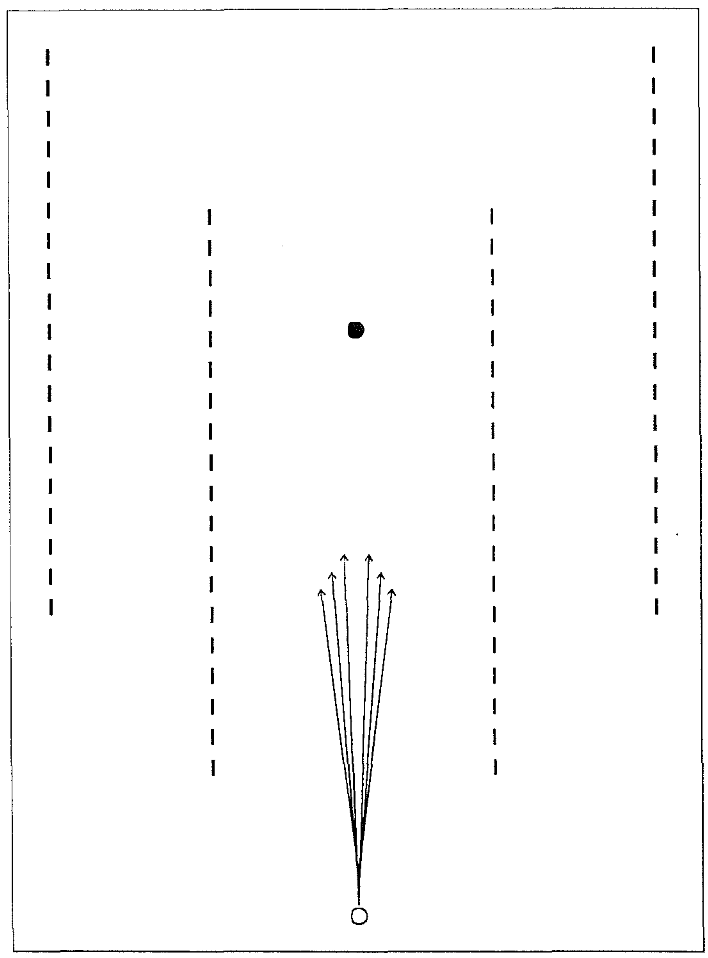

Figure 16. Schematic representation of the simple motion parallax environment viewed from above. The open circle represents the position of the simulated egolocus at trial onset. The filled circle represents the target. The six arrows represent the six possible initial deviations of the instantaneous movement direction from the target. Each of the four dashed lines represents a row of objects in the environment.

(3) Procedure

Instructions for Experiment 3 were only changed to reflect the increase in the number of experimental trials to 72. Otherwise, the procedure remained the same.

(a) Trial Presentation. Order of trial presentation was counterbalanced and randomised and with constraints, using a procedure described in detail in Appendix H.

II. RESULTS

(1) Preparation of Data and Exploratory Analysis

In total, 106 spikes were detected in the control vectors and subsequently replaced with zero entries. Visual analysis of the data revealed that the first subject had been presented with 6 identical events, while missing out on others. The error was traced to a software routine intended to improve the recording process. While the problem was solved for subsequent runs, the data for this particular subject were discarded. Consequently, subsequent analyses are based on data obtained from 12 subjects.

As in the previous two experiments, the data sets were terminated 60 samples before the deviation of the instantaneous direction of simulated self-motion from the target [heading error (θ)] reached a value of $|90|$ deg. Similarly, to compensate for the differential increase of $|\theta|$ until an appropriate control action was initiated the first 2 s of each trial were again excluded from analysis. This decision was supported by the magnitude of the reaction times, which were longer than in Experiment 1, but shorter than in Experiment 2 (Table 10).

Figure 17 indicates that the differences caused by the varying levels of $|\theta_i|$ had to a large extent already been compensated for by 2 s. Note that, as in Experiment 1, the effect of $|\theta_i|$ seemed to have a lasting effect on performance. This tendency seemed to be most pronounced at the highest level of GOFV.

Table 10. Mean times to onset of first correct control episode (in s) for Experiment 3.

	Mean Exp. 1	Mean Exp. 2	Mean Exp. 3	Std. Dev.	Cases
Target-only environment	2.05	12.18			
NMP environment			2.75	3.75	216
SMP environment			2.47	3.19	216
DMP environment	1.63	3.67	2.30	2.62	216
Random environment	1.44	1.73	1.42	1.52	216
$\lvert\theta_i\rvert$ = 2 deg	2.4	3.68	3.20	3.66	288
$\lvert\theta_i\rvert$ = 4 deg	1.47	2.74	2.08	3.04	288
$\lvert\theta_i\rvert$ = 6 deg	1.25	1.68	1.42	1.21	288
GOFV = 0.6 h/s		4.67	3.91	4.36	288
GOFV = 1.2 h/s		2.18	1.68	1.45	288
GOFV = 2.4 h/s		1.25	1.12	0.60	288

Note. θ_i = initial heading error, GOFV = global optical flow velocity.

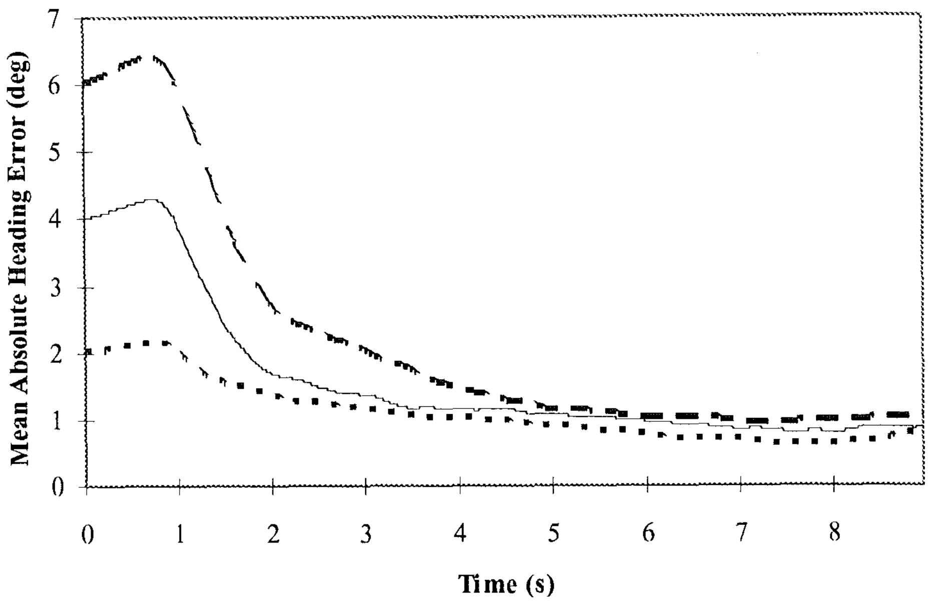

Figure 17. Mean absolute heading error (mean $|\theta|$) as a function of time and the three levels of $|\theta_i|$. Global optical flow velocity = 2.4 h/s (each point on a curve represents the average of 288 observations).

(2) <u>Analysis of the Extracted Variables</u>

The same variables as in the first two experiments were extracted from the raw data. A one-way MANOVA of all dependent variables of the form θ_i (left or right) yielded a nonsignificant main effect (Pillais $F(9, 854) = .8047$, ns). Consequently, the data were collapsed over left and right θ_i for all subsequent analyses.

As in Experiment 2, the main analysis took the form of a MANCOVA of all continuous dependent variables of the form Environment (NMP, SMP, DMP, or Random) x GOFV (0.6, 1.2, or 2.4 h/s), with $|\theta_i|$ and trial number as covariates. This yielded a significant multivariate within-cells regression effect (Pillais $F(18, 1686) = 14.8111$, $p<.001$) indicating that performance was significantly influenced by the covariates. The univariate tests showed that all dependent variables except for the number of incorrect control episodes were strongly affected (Appendix I, Table I-1). The associated regression coefficients (Table 11) indicate that performance deteriorated with increasing initial heading error, and improved with practice. This essentially replicates the results of Experiment 1 (cf. Table 1), even though the practice effect was a little smaller, while the effect of increasing initial heading error was larger.

Table 11. Regression coefficients for the dependent variables associated with each of the covariates.

	Covariates	
Dependent variables	$\lvert\theta_i\rvert$	Trial
Mean $\lvert\theta\rvert$ (deg)	.101**	-.003*
Mean $\lvert\theta\rvert$ during zero-control	.084**	-.003*
Mean $\lvert\theta\rvert$ at control conclusion	.104**	-.004**
.....Associated SD	.080**	-.003**
Mean $\lvert\theta\rvert$ at control initiation	.129**	-.006**
.....Associated SD	.109**	-.002*
Reaction time	-.445**	-.004
No. of correct control episodes	.162**	-.004
No. of incorrect control episodes	-.011	.000

Note. θ_i = initial heading error.

* p<.05, ** p<.001.

However, as mentioned in the previous section, visual inspection of the data indicated that the latter effect might have been restricted to the higher levels of GOFV. To test for this, a second partial MANCOVA of the form GOFV (0.6, 1.2, or 2.4 h/s) x $|\theta_i|$ (2, 4, or 6 deg) with trial number as a covariate was conducted. While the existence of a significant multivariate interaction effect (Pillais $F(36, 3396) = 2.86061$, $p<.001$) seemed to confirm those indications, univariate tests revealed that this effect was exclusively due to variations in reaction time ($F(4, 854) = 11.78521$, $p.<.001$). In addition, the relevant means (Figure 18) reveal that, while there was a large effect of $|\theta_i|$ on reaction times at the lowest level of GOFV, this effect actually *decreased* with increasing GOFV. Consequently, the use of $|\theta_i|$ as a covariate was judged to be appropriate.

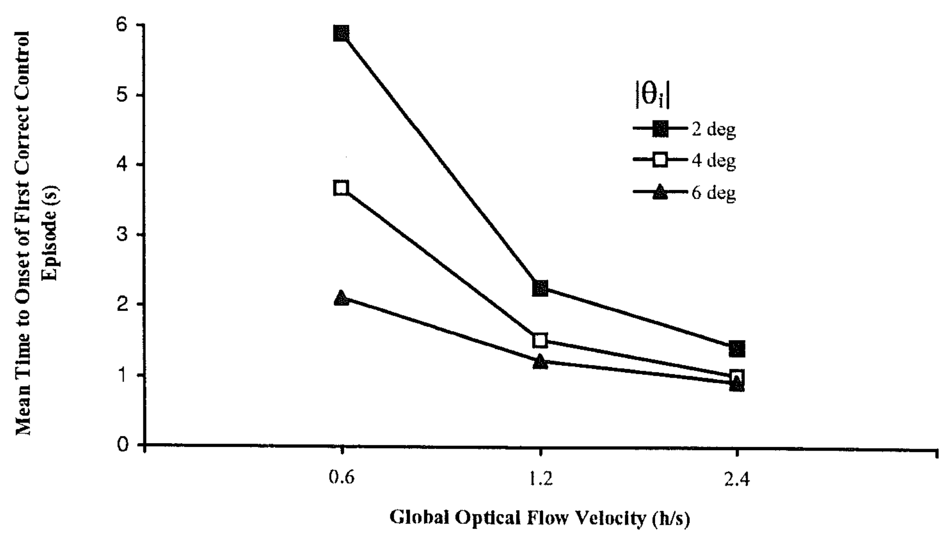

Figure 18. Mean time to first correct control episode as a function of global optical flow velocity and absolute initial heading error ($|\theta_i|$).

Together with the results of Experiments 1 and 2, where a similar effect - albeit nonsignificant in Experiment 2 - was observed, this outcome indicates that the effect of $|\theta_i|$ has longer lasting effects than previously suspected. To determine more precisely how long it took participants to compensate, the truncated data sets from each trial were divided into quarters. In addition, it was of interest to determine whether performance in the four quarters varied significantly over the three levels of GOFV, and over the four environments.

Only mean absolute heading error was extracted from the quartered data sets, since all other variables yielded an unacceptably large proportion of missing values in some of the cells. The results of an analysis of covariance (ANCOVA) of the form Quarter (1st, 2nd, 3rd, or 4th) x $|\theta_i|$ (2, 4, or 6 deg) x GOFV (0.6, 1.2, or 2.4 h/s) x Environment (NMP, SMP, DMP, or Random), with trial number as a covariate yielded significant effects for Quarter ($F(3, 3311) = 128.22$, $p<.001$), for the Quarter by $|\theta_i|$ interaction ($F(6, 3311) = 7.21$, $p<.001$), and for the Quarter by GOFV interaction ($F(6, 3311) = 3.36$, $p<.01$). Note that only effects involving Quarter are reported here, since the remaining effects are more appropriately dealt with by the main MANCOVA. The model explained 20.5 % of the total sum of squares (R^2), or 17.1 % of the variation in heading error (adjusted R^2). Figure 19 shows that there were large performance differences among the three levels of initial heading error in the first quarter of the truncated data sets. While by the second quarter the difference between the two higher levels of initial heading error had disappeared, it remained at a constant level between them and the lowest level of initial heading error over all four quarters of a trial. Overall, there was a relatively constant performance improvement from the first to the last quarter. A Bonferroni test of multiple comparisons confirmed these interpretations (df = 3311, p<.01; note that for each multiple comparison only the highest p-value that is smaller than 0.05 is reported).

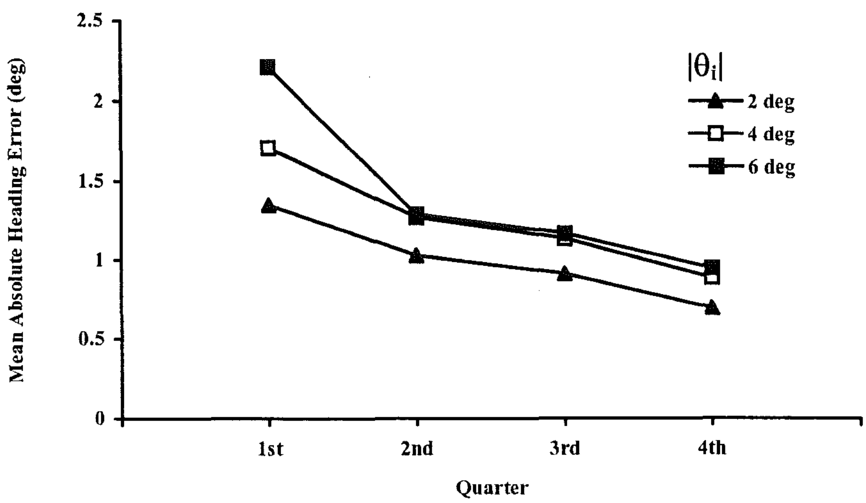

Figure 19. Mean absolute heading error (mean |θ|) as a function of trial duration and the three levels of absolute initial heading error ($|\theta_i|$).

Figure 20 again demonstrates the relatively constant improvement of performance over the four quarters of a trial, but also gives an insight into the effect of the GOFV manipulation over time. Note that although there is an overall difference between a GOFV of 0.6 h/s and the two higher levels, this difference is confined to the first three quarters of the trials[23].

[23] All discussed differences with the exception of the performance improvement from the second to the third quarter for the two higher levels of GOFV were significant (Bonferroni, df = 3311, p<.01).

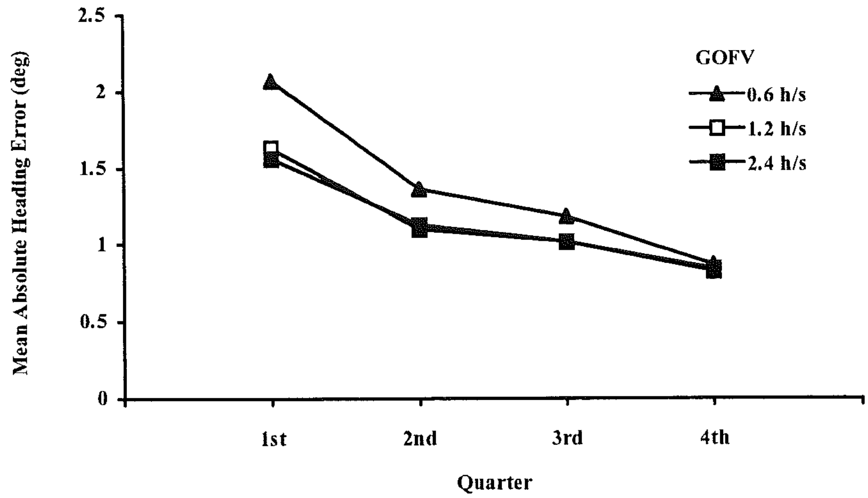

Figure 20. Mean absolute heading error (mean |θ|) as a function of trial duration, and the three levels of global optical flow velocity (GOFV).

The main MANCOVA also detected a significant multivariate Environment main effect (Pillais $F(27, 2532) = 2.6292$, p>.001). The associated univariate tests show that, with the exception of the number of both correct and incorrect control episodes, all dependent variables were strongly affected (all p<.001, for details see Appendix I, Table I-2). The results of a Bonferroni test of multiple comparisons indicate that performance in the Random environment was superior to that in all other environments (Table 12). There is some indication that performance in the NMP environment was poorer than in the two motion parallax environments. In addition, there is no indication of a difference between the performance in the SMP and the DMP environments.

Table 12. Differences between means for the four environments (for actual means see Appendix I, Table I-3).

	Environment					
Variable	NMP - SMP	NMP - DMP	NMP - Random	SMP - DMP	SMP - Random	DMP - Random
Mean |θ|	0.13*	0.139*	0.492***	0.009	0.362***	0.353***
Mean |θ| during zero-ctrl	0.139*	0.165*	0.527***	0.026	0.388***	0.362***
Mean |θ| at ctrl conclusion	0.081	0.048	0.337***	-0.034	0.256***	0.289***
.....Associated SD	0.054	0.012	0.195***	-0.042	0.141**	0.183***
Mean |θ| at ctrl initiation	0.133	0.108	0.538***	-0.024	0.406***	0.43***
.....Associated SD	0.057	0.049	0.284***	-0.008	0.228***	0.236***
Reaction time (s)	0.29	0.452*	1.332***	0.163	1.042***	0.88***

* p<.05, ** p<01, *** p <.001

The pattern of individual results for mean |θ| at the conclusion of control episodes shows few exceptions to the group effects (Table 13). As far as the significant effects are concerned, only the performance of subject 10 for the comparison between the NMP and the Random environment goes against the group effect. The situation is a little more complex with regard to the nonsignificant effects. A small number of subjects show a relatively substantial improvement from the NMP to the SMP environment (subject 9) and from the NMP to the DMP environment (subjects 4, 9, and 11). However, this does not seriously compromise a discussion of the group effects, since the corresponding group effects for mean |θ| and mean |θ| during zero-control episodes did reflect an overall improvement of performance.

Nevertheless, some subjects also showed a relatively substantial performance deterioration from the NMP to the DMP environment (subject 10), and from the SMP to the DMP environment (subjects 4, 8, and 10). This indicates that for these subjects there was a tendency for performance to be poorer in the DMP environment than in the remaining environments. Only subject 11 showed an improvement from the SMP to the DMP environment.

Table 13. Differences between means (mean |θ| at the conclusion of control episodes) for the four environments and the 12 subjects.

	Environment					
Subject	NMP - SMP	NMP - DMP	NMP - Random	SMP - DMP	SMP - Random	DMP - Random
1	-0.07	-0.21	0.43*	-0.14	0.50*	0.64*
2	-0.07	0.04	0.91*	0.11	0.98*	0.87*
3	-0.16	-0.05	0.24*	0.11	0.40*	0.29*
4	0.07	0.34	0.63*	-0.36	-0.07	0.29*
5	0.14	0.01	0.18*	-0.13	0.04	0.17*
6	0.14	0.20	0.37*	0.06	0.23*	0.17*
8	0.01	-0.09	0.31*	-0.19	0.21*	0.40*
9	0.35	0.41	0.27*	0.06	-0.08	-0.14
10	-0.14	-0.43	-0.27	-0.29	-0.13	0.16*
11	0.05	0.37	0.59*	0.32	0.54*	0.22*
12	-0.06	-0.01	0.16*	0.05	0.22*	0.17*
13	0.01	0.00	0.21*	-0.01	0.20*	0.21*

* Difference between means is in the same direction as the group effect.

The multivariate GOFV main effect also proved to be significant (Pillais $F(18, 1686) = 31.0397$, $p<.001$). The univariate tests show that all dependent variables were strongly affected (for details see Appendix I, Table I-4). The distribution of the means indicates that, much like in Experiment 2, performance improved most from 0.6 to 1.2 h/s, but only to a much lesser extent from 1.2 to 2.4 h/s (Table 14). In particular, the improvement from 1.2 to 2.4 h/s seemed confined to the two measures of variability and that of reaction time rather than those of performance in terms of heading error. In addition, the number of both correct and incorrect control episodes increased over all levels of GOFV.

Table 14. Means and differences between means for the three levels of GOFV for Experiment 3.

	Global optical flow velocity (h/s)					
Variable	0.6	1.2	2.4	0.6 - 1.2	0.6 - 2.4	1.2 - 2.4
Mean |θ|	1.32	1.12	1.11	0.20**	0.21**	0.01
Mean |θ| during zero-ctrl	1.29	1.09	1.08	0.20**	0.20**	0.00
Mean |θ| at ctrl conclusion	1.13	0.93	0.88	0.20**	0.25***	0.05
.....Associated SD	0.89	0.68	0.54	0.20***	0.35***	0.14**
Mean |θ| at ctrl initiation	1.99	1.68	1.54	0.31***	0.45***	0.14
.....Associated SD	1.16	0.88	0.66	0.28***	0.50***	0.22***
Reaction time (s)	3.90	1.68	1.12	2.22***	2.78***	0.56**
No. of correct ctrls	5.92	4.68	3.73	1.24***	2.19***	0.95***
No. of incorrect ctrls	0.43	0.24	0.12	0.19***	0.32***	0.13**

* p<.05, ** p<.01, *** p<.001

The individual results for mean $|\theta|$ at the conclusion of control episodes are represented in Table 15. They indicate that the majority of subjects (1, 3, 4, 5, 6, 8, 10, 11, 12, and 13) mirror the group effects. The performance of subject 2 remained relatively constant, and only subject 9 showed a slight performance deterioration from a GOFV of 0.6 to 1.2 h/s.

Table 15. Means and differences between means (mean $|\theta|$ at the conclusion of control episodes) for the three levels of GOFV and the 12 subjects of Experiment 3.

	Global optical flow velocity (h/s)					
Subject	0.6	1.2	2.4	0.6 - 1.2	0.6 - 2.4	1.2 - 2.4
1	1.36	1.16	1.03	0.20*	0.06*	0.13
2	1.39	1.42	1.41	-0.03	-0.02	0.01
3	1.20	0.67	0.65	0.53*	0.55*	0.02
4	1.81	1.33	1.28	0.48*	0.53*	0.05
5	1.20	0.89	0.70	0.31*	0.50*	0.19
6	0.60	0.44	0.59	0.16*	0.01*	-0.15
8	1.07	0.91	0.96	0.16*	0.11*	-0.05
9	1.07	1.13	0.94	-0.06	0.13*	0.19
10	0.95	0.89	0.71	0.06*	0.24*	0.18
11	1.71	1.41	1.32	0.30*	0.39*	0.09
12	0.48	0.42	0.47	0.06*	0.01*	-0.05
13	0.71	0.47	0.49	0.24*	0.22*	-0.02

* Difference between means is in the same direction as the group effect.

Finally, a significant Environment by GOFV interaction was detected (Pillais $F(54, 5082) = 1.8115$, $p<.001$). Examination of the univariate test results indicated that this was mainly due to differences in reaction time ($F(6, 850) = 5.5953$, p.<001). The distribution of the relevant means suggests that although there were large differences in reaction time at the lowest level of GOFV among the four environments, these differences dissipated with increasing GOFV (Figure 21). In addition, the improvement of mean reaction time with increasing GOFV in all four environments was subject to a floor effect at around 2.4 h/s. The results of a Bonferroni test of multiple comparisons confirmed these interpretations: While there were significant differences between all means at the GOFV of 0.6 h/s (df = 850, $p<.05$), there were none at the two higher levels of GOFV. Similarly, while

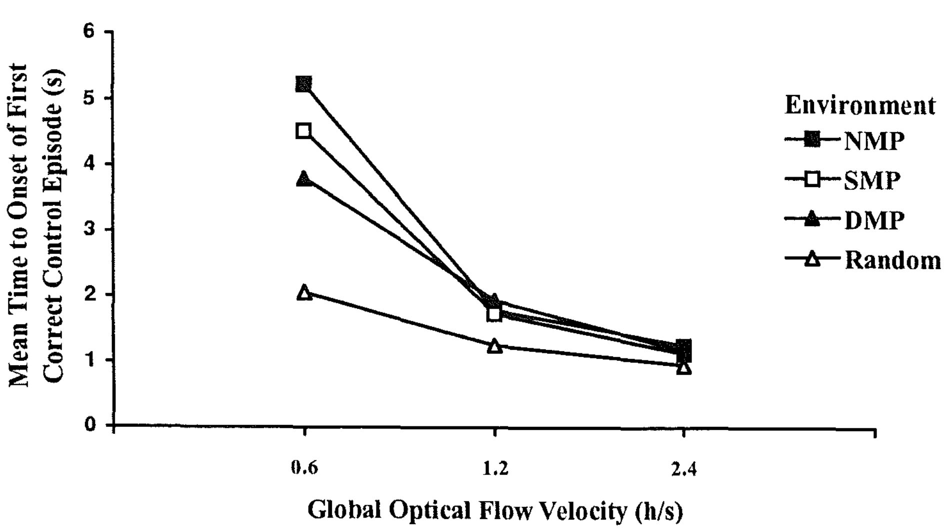

Figure 21. Mean time to onset of first correct control episode as a function of global optical flow velocity and environment.

performance improved from the lowest to the medium level of GOFV in all environments (df = 850, $p<.05$), the DMP environment was the only one that showed a reduction in mean reaction time from the medium to the high level of GOFV (df = 850, $p<.05$).

III. DISCUSSION

(1) Effect of Initial Conditions

One of the aims of Experiment 3 was to allow a closer examination of the failure of Experiment 2 to find a significant influence of initial absolute heading error on performance, even though it was detected in Experiment 1. Two alternative explanations were proposed. One involved the differential impact of the chosen level of gain for small and large values of $|\theta_i|$, the other implicated the smaller cell size of Experiment 2. Results of Experiment 3 clearly indicate that the latter explanation holds true. Much like in Experiment 1, $|\theta_i|$ exerted a significant effect on performance (a similar result was observed with regard to the practice effect). Closer examination of this effect revealed a number of interesting findings. (a) The existence of a performance difference between all three levels of $|\theta_i|$ during the first quarter of a trial, together with the fact that the difference between $|\theta_i|$ of 4 and 6 deg had disappeared by the second quarter suggests that the elimination of the first 2 s of each trial may not have been quite enough to allow subjects to compensate for differing levels of initial heading error (keep in mind that the first quarter excludes the first 2 s of each trial). However, the persistence of a difference between a $|\theta_i|$ of 2 deg and the other two conditions throughout the entire trial indicates that this does not fully explain the effect of $|\theta_i|$ on performance. It seems that, as suggested previously, the level of initial heading error to some extent influenced the establishment of some visual criterion for acceptable performance. This 'perceptual anchoring' effect may operate only at levels of $|\theta_i|$ that are relatively close to the perceptual threshold. (b) Performance in terms of mean $|\theta|$ showed a relatively constant improvement over the four quarters of a trial. It seems that the perception of heading with respect to a target becomes more accurate with decreasing *distance* to that target. The fact that this was not simply a function of the *time* spent 'finetuning' the heading is demonstrated by the fact that there were virtually no differences over the four quarters between the high and medium levels of GOFV, despite the fact that a quarter of a trial with a GOFV of 2.4 h/s lasted only half as long as one with a GOFV of 1.2 h/s. As a matter of fact, the exact opposite effect was observed for the comparison between trials with a GOFV of 0.6 h/s and the other two conditions, i.e., despite the fact that more time was available for fine control during the lowest level of GOFV, performance was actually worse. Similar comments can be made with regard to the possibility of increasing human sensitivity to heading error with decreasing time to contact (Lee, 1976). Although in this instance the decreasing difference between the GOFV of 0.6 h/s and the two higher levels over the four quarters would be in the predicted direction, the fact that there is none between the two higher levels of GOFV does not fit such an hypothesis. Of course, since trial length covaried with GOFV, these conclusions are relatively weak and would need further corroborating evidence to achieve reasonable credibility. Nevertheless, as a first indication they suffice.

Accepting for the moment that accuracy of heading perception does increase with decreasing distance to the target, the question arises why this might be the case. There is a relatively straight forward and intuitive explanation of this effect: Consider the following 'separation ratio':

$$\sigma = 1-(d_{A\text{-}NO}/d_{A\text{-}FO}), \tag{3}$$

where $d_{A\text{-}NO}$ is the distance between the actor and a near object somewhere in the environment, and $d_{A\text{-}FO}$ is the distance between the actor and a farther object in approximately the same vertical sector of the optic array. Setting $d_{A\text{-}NO}$ to unit measure, this expression will always have a value of between zero and one and will increase as the objects are approached. Consequently, it will index an increase in the differential optical motion of the objects. In contrast, as the value of σ approaches zero, this differential motion decreases, and is at a minimum when both objects are equidistant from the moving point of observation[24]. If actors are sensitive to the information made available by this kind of relative motion, performance would be expected to improve in the observed fashion. In addition, it would be expected to be optimally informative if both objects lay along the same vertical plane of sight, and to become increasingly noninformative as the horizontal optical angle between them increases (Gibson et al., 1955). The effect of increasing σ on optical motion is not

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[24] The term 'separation ratio' was chosen to reflect its direct dependence on the separation in distance between the observer and two objects. The greek symbol σ (sigma) follows from the first letter of the term. Note further that Rieger & Lawton (1985) first suggested that heading judgments might improve "as the ratio of the separation of ... [two fronto-parallel] planes over their distance from the observer increases" (p. 359).

obvious for the case of self-motion directly towards a target; it is probably most apparent when an observer does *not* head directly towards the intended target, i.e., when correction is needed. Consider two points at different distances from a moving observer (Figure 22). The observer begins to move at some angle to the nearest object (the target). With continued movement, the optical angle enclosed by the two objects begins to grow. This is the (motion parallax) information that tells the observer to start correcting the path of self-motion towards the target, i.e., into a direction that slows the growth of this angle. Note that if the observer starts out at a position closer to the two objects (at a position where σ is greater), this angle grows

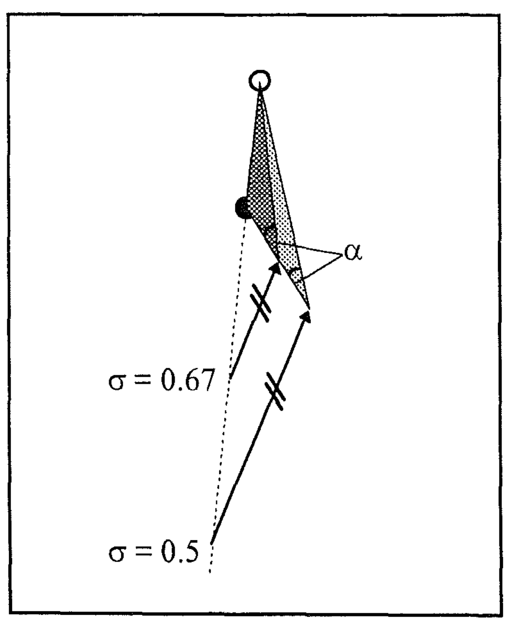

Figure 22. Two situations where an observer moves toward the right side of a target (filled circle). Note that heading error at the beginning and the end of the two vectors is identical. Similarly, the optical angles enclosed by the two objects at the beginning of each vector are identical (0 deg). Nevertheless, at the end of the vector associated with the larger separation ratio (σ), this angle (α) has become considerably larger than the one associated with the smaller ratio.

at a more rapid rate, despite the fact that heading errors for both movement vectors are identical. The net result of this faster optical change is that heading information becomes more readily available as an object is approached. To be sure, this type of information is better at specifying that one is going to miss a target rather than hit it, but in terms of functionality for the detection of heading error, it is simply the reverse side of the same coin.

(2) Global Optical Flow Velocity

Results of Experiment 3 with respect to the measures of performance in terms of absolute heading error replicated those of Experiment 2. Performance improved from GOFV of 0.6 to 1.2 h/s, but remained constant from 1.2 to 2.4 h/s. This strengthens support for the position that increased speed of simulated self-motion leads to more accurate heading perception, and that this beneficial effect is subject to a floor effect in the top GOFV range of human bipedal locomotion. However, the fact that not all of the dependent variables showed the floor effect needs to be addressed. In particular, the number of both correct and incorrect control episodes, the two measures of variability, and time to onset of first correct control episode still showed significant differences between the two higher levels of GOFV. It is suggested that the first two of these variables are dependent on the duration of a trial as well as on the level of GOFV. In particular, during lower levels of GOFV there was simply more time available to engage in both correct and incorrect control episodes (10, 20, and 40 s for GOFV of 0.6, 1.2, and 2.4 h/s, respectively). The interpretation of the continuing decrease of the two measures of variability is a bit more problematic. It is useful to note that one of the measures (mean standard deviation at the conclusion of control episodes) did not show this pattern of results in

Experiment 2. This raises some doubt with regard to the stability of the difference observed in Experiment 3. On the other hand, the continuing decrease of the second measure (mean standard deviation at the initiation of control episodes) did replicate results of Experiment 2, and is interpreted to indicate a continuing, but diminishing improvement of human sensitivity to heading information with GOFV. In other words, while mean $|\theta|$ at the initiation of control episodes only showed a trend towards improvement, the significant decrease in the associated mean standard deviation shows that the cluster of responses around this mean became tighter at the highest level of GOFV. This represents one more indication that performance improvement was indeed levelling out at a GOFV of 2.4 h/s. Similar comments can be made with regard to reaction time. Improvement from a GOFV of 1.2 to 2.4 h/s was (a) confined to the DMP environment and (b) smaller than from a GOFV of 0.6 to 1.2 h/s. In contrast, all environments yielded improved reaction times from a GOFV of 0.6 to 1.2 h/s.

A recent investigation by Vishton and Cutting (1995) has looked into whether the spatial or the temporal aspects of optical motion during changes in GOFV determined the accuracy of heading judgments. They conducted three experiments, during each of which either velocity, the duration of a trial, or the covered distance was held constant. Significant performance differences were only observed in the first two scenarios. They interpreted this result to indicate that the displacement field rather than the velocity field or the trial duration accounted for any performance differences found during manipulations of GOFV[25]. Results of the present experiment do not support this interpretation, i.e., although the covered distance was always the same, and consequently independent of the level of GOFV, performance still improved with increasing GOFV. There are two methodological differences between the two studies that may help to explain this discrepancy. (a) The event durations used in the present investigation were considerably longer (ranging between 10 and 40 s, as opposed to 0.5 and 4 s in Vishton and Cutting's study). It is possible that for extremely brief events displacement information is more important, while during longer, ecologically more valid events velocity information is used as well, or instead. (b) The present investigation used an active control paradigm, while Vishton and Cutting used a passive observation paradigm. Research evidence described earlier (see p. 20) indicates that performance on active and passive observation tasks may differ (see also Wann, Rushton & Lee, 1995). On the other hand, an active control study by Owen and Wolpert (1987) found *decreased* sensitivity to change in altitude with increasing GOFV. This would appear to open the possibility that people use different types of information depending on the task demands of a situation. In particular, since descent detection and target approach are quite different tasks, it is possible that increasing GOFV has a negative effect on the former and a positive effect on the latter. During flight changing GOFV specifies (a) changing eyeheight or (b) changing forward velocity. Consequently it is not uniquely informative about altitude. On the other hand, if eyeheight is constant during forward self-motion (as is

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[25] In response, a subsequent paper encourages to adopt the term 'differential motion *displacement*' (Cutting et al., 1995) instead of the earlier introduced term 'differential motion parallax' (Cutting, 1986).

the case when walking or driving) increasing GOFV specifies increasing forward velocity. This in turn specifies more adverse consequences of a possible collision. More importantly, differential (horizontal) optical motion then becomes more rapid, i.e., it becomes easier to detect. To borrow a sentence from Owen and Wolpert (1987, p. 153): "What is easier to detect, is easier to control". Owen and Wolpert simulated flight over an endless plane with square fields. Differential (vertical) optical motion is only present to a limited extent during level flight over such terrain. The altitude-control equivalent of the present study would be to give subjects control over the path slope during simulated helicopter descent, and to ask them to approach a landing pad on the ground in a straight line. It is possible that in this case increasing GOFV has a beneficial effect on performance. As a 'terrestrial' control condition, the NDM environment of Experiment 4 could be used, where the initial approach angle to the row of objects would have to be the same as the initial path slopes in the path-slope control condition. Conversely, a terrestrial control condition for Owen and Wolpert's descent detection task could be created. Again, the basic setup of the NDM environment of Experiment 4 could be used. In this case level self-motion parallel to a row of objects would be simulated. The task would be to keep the distance to the row constant in the presence of lateral disturbances. It is possible that in this case performance would deteriorate with increasing GOFV.

Be that as it may, results of the present experiment indicate that during events of level target approach (a) there is a beneficial effect of increased GOFV, (b) performance optimizes in the velocity range of human bipedal locomotion, and (c) velocity field information rather than displacement information is used.

A final finding with respect to the effect of GOFV is that its impact decreased with decreasing distance to the target. As pointed out in the previous section, relative optical motion becomes more pronounced as σ increases. If this in turn leads to an improvement in performance, perception-action limits are approached. As a consequence, the beneficial effect of increased GOFV would be expected to have less room to become apparent.

(3) Simple and Differential Motion Parallax

The performance improvement from the DMP to the Random environment replicates the corresponding effect observed in Experiment 2. Together with results from Experiment 1, where there was no such difference, this strengthens the conclusion that target drift (operational in Experiment 1, but not in Experiments 2 and 3) is a type of optical information that facilitates the perception and control of heading during simulated events that also contain DMP information. Similarly, the fact that the number of both correct and incorrect control episodes did not vary among the four environments replicated results from Experiment 2. Together with results from the comparison between Experiments 1 and 2, where the number of correct control episodes was found to be inversely related to performance in terms of heading error, and from Experiment 1, where it was directly related to heading error, this result supports the conclusion that the number of control episodes is not a particularly reliable indicator of perception-action performance in a simulated heading task.

The four experimental environments were designed to allow an evaluation of the functionality of motion parallax information in general, and DMP information in particular, for the task of approaching a target. The NMP environment was specifically designed not to include any motion parallax information in the traditional sense. In other words, there were never two or more objects that crossed the same plane of sight at the same time. Nevertheless, performance in that environment was very good (mean $|\theta|$ at the conclusion of control episodes = 1.1 deg). Much like in Experiment 2, this level of performance is within the required wayfinding accuracy proposed by Cutting et al. (1992) of around 1.86 deg for a GOVF of 2.4 h/s. Three types of information can be identified that might have been used by subjects to control the simulated path of self-motion in this environment. (a) Objects other than the target were arranged along a straight line on each side of, and approximately parallel to, the direction of simulated travel. The optical projection of the extensions of these two lines to the horizon form what has been termed an 'optical splay angle' (R. Warren, 1982). Calvert (1954) was the first to suspect that this optical variable, or its temporal derivative, might be used to control the flight path of an aircraft through a 90 deg turn from base leg to final leg of a landing approach. Loomis and colleagues recently begun to investigate this possibility (Loomis & Beall, 1992; Beall & Loomis, 1995b), as well as the role this variable might play in steering a vehicle along a straight road (Beall & Loomis, 1995a). In addition, optical splay has been found to be a functional type of information for the perception of altitude both during simulated forward self-motion (e.g., Flach, Hagen & Larish, 1992; Johnson, Tsang, Bennett & Phatak, 1989; Levison & R. Warren, 1984; R. Warren, 1988), and during simulated hover (Owen, Florence-Bennett & Frey, 1994). (b) The display contained exclusively diagonal edges. Consequently, the horizontal as well as the vertical components of global optical flow were present during simulated self-motion. The optical motion of either component specifies the focus of radial outflow in the corresponding plane, at least during linear translation. Subjects may have determined the location of the focus during straight line travel (zero-control episodes), and subsequently corrected the path such that it coincided with the target. (c) Subjects may have controlled the simulation such that they attempted to minimize the rotational flow of objects around the target. If it is nulled, simulated self-motion is directly towards the target. However likely or unlikely these three explanations may be, unless the addition / subtraction of information led to different control strategies, they hold equally for the other experimental environments. Consequently they cannot account for the performance differences observed between the environments and can therefore for the moment be disregarded.

The addition of a second row of objects on either side of the moving point of observation yielded a display containing 'simple' motion parallax (SMP) information. Throughout simulated self-motion in that environment, there were no more than two discontinuities along many vertical planes of sight, passing by at different optical velocities. The difference in velocities between two such discontinuities yields additional heading information as they cross the same vertical neighbourhood (Rieger & Lawton, 1985). Performance was expected to be facilitated by this information. As it turned out, two of the dependent measures did show a difference in the expected direction. In particular, both mean $|\theta|$ and mean $|\theta|$ during zero-control improved by approximately 0.13 deg (10 %). The fact that this is a relatively small performance improvement, and that none of the other measures showed the effect indicates that SMP information as defined above adds only little functionality to a simulation that already carries splay and other global flow information.

According to Cutting (1986; Cutting et al., 1992) there must be at least three objects at differing distances from the moving point of observation along a vertical plane of sight for DMP information to become available. The addition of a third row of objects on each side of, and parallel to, the line connecting the initial point of observation and the target resulted in an environment in which DMP information became available throughout the simulated approach to the target. It was expected that if people use this kind of information to guide approach to a target, performance would improve. However, this was not the case. The perhaps most striking finding of Experiment 3 was the complete lack of a performance difference between the SMP and the DMP environments. Since performance in absolute terms was nevertheless very good (mean $|\theta|$ at the conclusion of control episodes $\cong$ 1 deg) it might be argued that there simply was almost no room for improvement. However, the difference between the Random environment and the two motion parallax environments demonstrates that this was not the case. Clearly, an alternative explanation is needed.

Cutting (1986) showed that people can judge their heading relatively accurately using DMP information, if it is the only type of information available. However, due to the type of simulation and/or the use of a passive observation paradigm he could not test whether SMP information might have yielded similar results. More specifically, he simulated linear self-motion to the side of a group of objects arranged along three lines that were approximately perpendicular to the direction of self-motion and located at different distances from the observer. Since he was interested in isolating DMP information, the objects he simulated were vertical 'bars' that subtended the full height of the display. This ensured that no information other than relative horizontal optical motion of the simulated objects became available. There was no expansion of the bars, no (dis)occlusion, and since the top and bottom ends of the bars were not visible, no vertical components of optical flow. The simulation was centered on a bar located on the line in the middle distance, and as the moving point of observation moved to one side of it, the other bars appeared to move horizontally in relation to that focal bar. Under most circumstances, the fastest optical motion in such a simulation is carried by those bars that are closest to the observer. That optical motion will be in a direction opposite that of the direction of self-motion. In other words, if the fastest moving bar(s) appear to move left, the direction of self-motion is to the right of the focal bar. However, a test of the functionality of SMP information would involve the arranging of the bars along only *two* lines oriented at right angles to the direction of simulated self-motion. Using the same type of simulation where one of the bars remains in the center of the display during simulated self-motion, the problem would then have two equally likely solutions: The focal bar is either situated on the line that is the closer one to the observer, in which case the optical motion of the remaining bars is in the same direction as the direction of self-motion, or on the line that is the one further away from the observer, in which case the optical motion of the remaining bars is in the opposite direction as the direction of self-motion. As a consequence, performance on passive observation trials would necessarily be at chance level. Note that during an actively controlled trial only the first response would be expected to be at chance level. Once a control episode is initiated, the consequent change in the optical information specifies the solution to the problem[26]. The second consideration is that Cutting's simulation only contained the horizontal components of optical flow. If the bars had not subtended the full height of the display, the relative vertical optical motion of the top and/or bottom ends of the bars might have provided information

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[26] If the control action is to the right and the optical motion of the bars speeds up, the focal bar is on the line that is further away from the observer, and the direction of self-motion is to the right of the focal bar. If it slows down, the focal bar is on the line that is closer to the observer, and the direction of self-motion is to the left of the focal bar.

specifying their distance from the observer. It is here suggested that optical events that are neither actively controlled nor contain vertical as well as horizontal components of optical flow are extremely rare and artificial. To reiterate, Cutting (1986) has shown that *if* these conditions apply, humans can still determine their heading with relatively high accuracy. However, this does not mean that humans use this type of information under different circumstances. Although Cutting used his findings to substantiate the claim that DMP information "is likely to be the best information for the task [of direction finding] at normal speeds" (p. 217), he rightly added on the same page that it is by no means the only, or even a necessary, one.

In a later paper Cutting et al. (1992) employed simulations of a kind that did include both horizontal and vertical components of optical flow. They simulated linear self-motion to one side of a group of 'trees', each consisting of a long vertical line, crossed by a smaller horizontal one at its midpoint at eyeheight. The simulated trees expanded in so far as the lines appeared to grow longer as they were approached, but not wider. Both tops and bottoms of the trees were visible throughout the trials, conveying the vertical component of optical flow. Nevertheless, although the authors attempted to show post-hoc that performance dropped off in trials during which DMP 'failed' (i.e., trials during which objects closer to the observer than the focal tree had smaller retinal velocities than those that were further away), they did not provide a direct, controlled test of this suggestion. More importantly, they did not investigate the possibility that SMP information alone might go some way to explaining the observed results. It is here suggested that results of the present experiment indicate that in ecologically valid environments peripheral DMP information adds no functionality to an actively controlled simulation that already contains peripheral SMP information. However, two important qualifications need to be investigated: First, all of the experimental environments in the present investigation with exception of the Random environment had no environmental clutter in the central part of the display. All simulated objects other than the target were situated in the peripheral display. It is possible that motion parallax information in general, and DMP information in particular becomes more important for human actors as it becomes available in the immediate vicinity of a target that is to be approached. Andersen (1986) has reviewed evidence to the effect that human observers may acquire different kinds of information from central and peripheral visual fields. For example, in the context of visually simulated self-motion, central, but not peripheral visual field information may require the presence of depth variations for vection to occur (Andersen & Braunstein, 1985). Similarly, in investigating the type of information used in the control of stance, Stoffregen (1985) found that the retinal periphery is less sensitive to radially expanding flow than the fovea, while Lishman and Lee (1973) and Lee and Lishman (1975) showed that lamellar peripheral flow (moving room) will induce compensatory body sway. Wolpert (1990) rightly emphasized the importance of a distinction between field of view (the complete sector of the optic array available to the eye) and retinal field (the area of the retina exposed to a particular sector of the optic array). He presented a review of empirical evidence demonstrating that both may be important factors in determining the perception and control of self-motion. Finally, there are three studies that dealt specifically with the role of central versus peripheral vision in the perception of heading. In controlling for field of view but not for retinal stimulation, R. Warren (1976) showed that although observers could detect the direction of their heading to some extent whether or not the FRO was in the field of view, performance deteriorated somewhat with increasing deviation of FRO from the center of the display. This result was replicated by findings from Experiment 1. More recently Crowell and Banks (1993) and W. H. Warren and Kurtz (1992) presented observers with simulated linear self-motion through random dot clouds, controlling for both retinal area of stimulation and field of view. Results of those studies indicated greater accuracy of heading judgments with radial rather than lamellar flow (regardless of the part of the retina being stimulated), and, to a lesser extent, greater foveal sensitivity to radial flow. Taken together, these findings suggest that both the type of information present in the field of view and, to a lesser extent, the area of the retina exposed to this information may influence perceptual performance. Although the direction of gaze, and consequently the content of the retinal field was not controlled for in the present experiment, there were indications that subjects spent the largest amount of time during a trial looking directly at the target. In particular, during practice trials a number of the subjects asked for the brightness to be turned down, since the yellow target tended to leave a distinct afterimage. This would only occur as a consequence of relatively constant fixation of the target. It is thus possible that motion parallax information becomes more important as it occurs more centrally in the visual field[27]. This possibility will be examined in more detail in the next experiment.

The second qualification concerns the relatively narrow working definition of motion parallax used so far. Accordingly, motion parallax is said to occur if, and only

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[27] Even though motion parallax information is less obvious when looking in the direction of travel during linear self-motion than when looking to the side of it (Gibson et al., 1955, p. 373), its functionality for controlling self-motion increases. A motion parallax rule to straight line approach to a target might be to correct for all differential optical velocities in the intended direction of self-motion.

if, during simulated self-motion at least two objects at different distances from the moving point of observation cross the same vertical plane of sight at the same time. While such a narrow definition is necessary for certain decompositional models that rely on differential velocity vectors at (dis)occluding edges (Longuet-Higgins & Prazdny, 1980), Rieger and Lawton (1985) have generalized this idea to spatially more separated flow vectors. Although it is unclear if and to what extent increasing optical separation might affect performance in terms of the detection of the direction of self-motion, the fact remains that all experimental environments that have been used so far - with the exception of the Target-only environment - contained optically separated differential optical motion. Previously I have argued that the NMP environment contained three types of information that specify the direction of simulated self-motion (splay angle, horizontal and vertical components of optical flow, and rotational optical motion around the target). If the definition of motion parallax is extended to include differential motion whose vectors are not only optically separated from each other in the vertical, but also in the horizontal dimension, there are four types. Even if one assumes that the functionality of that information decreases as optical separation increases, the initial horizontal optical angle between the target and the laterally closest other object in the display was relatively small at around 4 deg. Since the relative difference between the optical motion of that object and the target is directly available (the horizontal velocity vector of the target is effectively cancelled by the simulation), it can be argued that all of the displays contained some form of motion parallax[28]. This argument could be extended

[28] This links back nicely to the suggestion that differential optical motion, and, as a result, its functionality for the perception and control of heading may increase with σ (Equation 3, p. 97).

to deny that the preceding experimental manipulations amounted to a fair test of the functionality of simple as well as differential motion parallax information for the perception and control of heading.

To test for the functionality of motion parallax information for the perception and control of heading while taking into account the considerations raised in the previous paragraphs, a number of new environments were designed. First, an environment in which simulated self-motion only makes minimal differential optical motion available had to be created. In addition, heading information made available by the rate of change of an optical splay angle had to be eliminated. A row of objects placed along a line perpendicular to the line connecting the initial point of observation and the target fulfils this requirement. The target was also positioned on this line. Evidence from passive observation studies indicates that the accuracy of heading judgments during approach to a plane perpendicular to the axis of linear translation is very poor (Llewellyn, 1971; Johnston et al., 1973; Regan & Beverley, 1982), especially if target drift is eliminated from the displays. Consequently, performance in this environment is expected to be at a level that does not support goal directed self-motion. To make SMP available, a second row of objects, parallel to the first was added to the environment. In one environment the second row was placed in front of, in another environment the row was placed behind the row containing the target. Performance was expected to improve to an extent comparable to the one observed in the SMP environment of Experiment 3. The addition of a third row of objects, parallel to the first two made DMP information available at the moving point of observation. There were three DMP environments. In the first the two additional rows of objects were positioned in front of, in the second on either side of, and in the third behind, the one containing the target. If performance improves, DMP must provide functional information above and beyond that provided by SMP. In addition, comparisons between the two SMP environments and the three DMP environments will allow an evaluation of the effect of the σ. Also, the inclusion of the Random environment will allow a comparison of performance in the motion parallax environments with that in a relatively cluttered environment. Finally, to evaluate whether human actors use information provided by differential optical motion as it becomes available in the peripheral field of view, peripheral information will be blanked out on half the trials.

CHAPTER V

EXPERIMENT 4

I. METHOD

(1) Participants

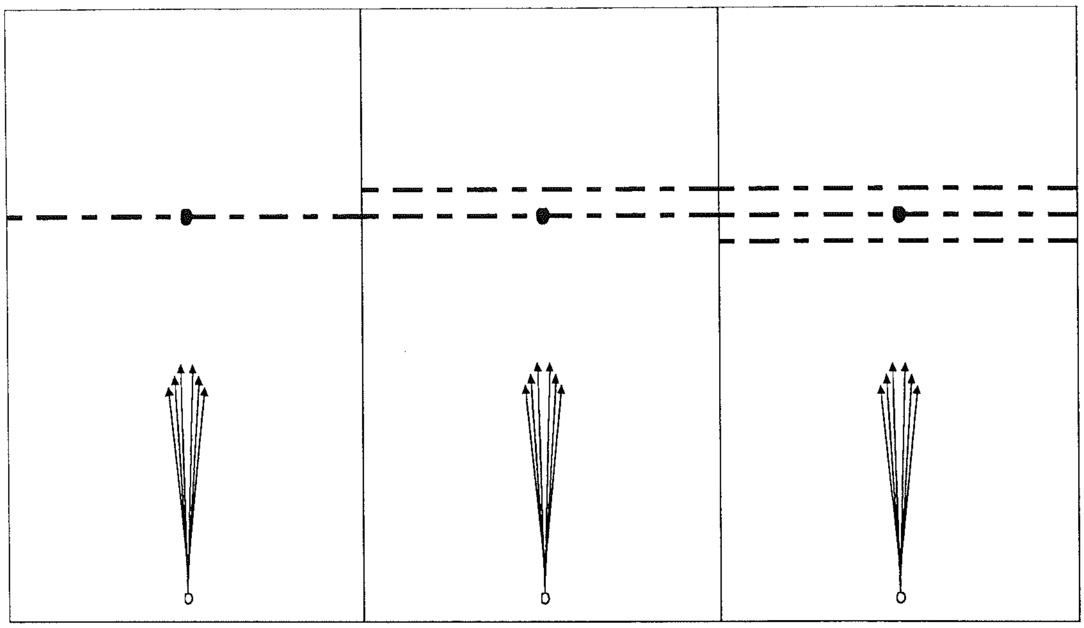
Twelve of the participants of Experiment 3 also took part in Experiment 4. One additional male participant aged 23 was recruited.

(2) Apparatus

(a) Kinematics. As in the previous three experiments, eyeheight (h) was kept constant, and as in Experiment 1, global optical flow velocity was set at 1.2 h/s. Consequently, the target was passed after approximately 20 s. The simulation technique was the same as in Experiments 2 and 3.

(b) Environments. To find answers to the questions raised in the discussion of Experiment 3, six environments were introduced in addition to the Random environment. All of those additional environments differed to those used in the previous two experiments only with respect to the number and placement of simulated objects (other than the target). A brief description of each environment is presented below. For a more detailed description, refer to Appendix A.

No Differential Motion (NDM) Environment. The NDM environment was designed to allow the evaluation of task performance in an environment where no differential optical motion was available throughout any trial. This was achieved by placing one row of objects perpendicular to a line connecting the initial point of observation and the target (Figure 23a).



a b c

Figure 23. Schematic representation of (a) the NDM environment, (b) one of the SMP environments, and (c) one of the DMP environments, viewed from above. The open circle represents the position of the simulated egolocus at trial onset. The filled circle represents the target. The six arrows represent the six possible initial deviations of the instantaneous movement direction from the target. The dashed lines represent rows of objects in the environment.

Simple Motion Parallax (SMP) Environments. To make simple motion parallax (SMP) information available, a second row of objects was added to the NDM environment, parallel to the first (Figure 23b). The location of the row not containing the target was either in front of or behind the target, resulting in two SMP environments, SMPF and SMPB, respectively.

Differential Motion Parallax (DMP) Environments. The addition of a third parallel row of objects to each of the SMP environments ensured that DMP information became available for the duration of a trial (Figure 23c). This third row was either placed in front of or behind the two already existing rows, resulting in a total of three DMP environments, DMPF, DMPM, and DMPB, respectively.

(c) Field of View. In total there were now seven distinct experimental environments (NDM, SMPF, SMPB, DMPF, DMPM, DMPB, and Random). This number was then doubled by blocking the peripheral field of view on half of the trials with two black rectangular polygons. This resulted in a narrowing of the simulated field of view by two thirds, i.e., the horizontal optical angle enclosed by the simulation was effectively reduced from 30.8 to 10.3 deg (see Figure 24 for an example).

(3) Procedure

The instructions used for Experiment 3 were only changed to reflect the increase in the number of experimental trials to 84 and the introduction of trials with a reduced field of view. Both demonstration and practice trials were adjusted accordingly. Otherwise, the procedure remained the same.

(a) Trial Presentation. Order of trial presentation was randomised with constraints, using the procedure described in detail in Appendix J.

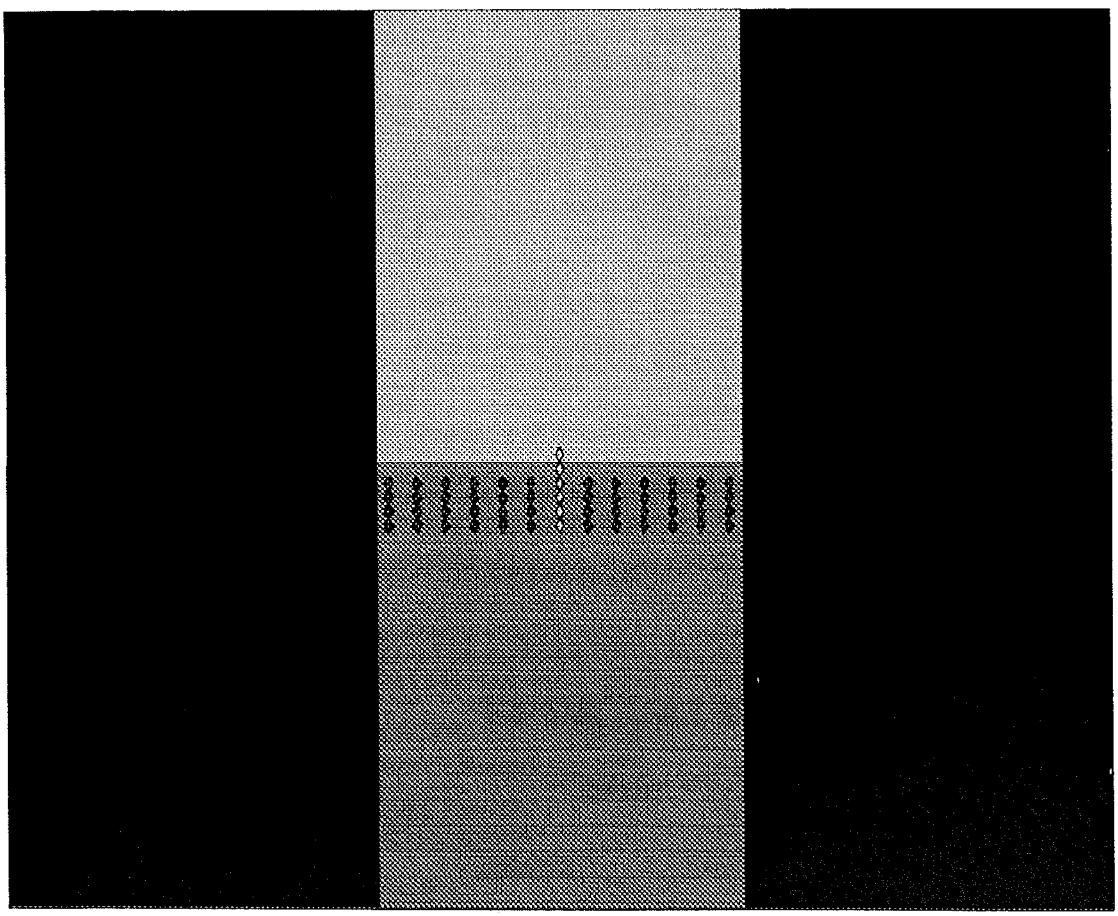


Figure 24. View of the NDM environment as it appeared on the display at the onset of a trial with peripheral field of view blocking in place.

II. RESULTS

(1) Preparation of Data and Exploratory Analysis

Prior to further analysis 79 spikes were replaced with zero entries. Some of the data files of two of the subjects (among them the newly recruited one) were corrupted during a compression process. Consequently, all subsequent analyses are based on data of the remaining eleven participants.

Data sets were again truncated 60 samples before heading error (θ) reached a value of $|90|$ deg. Reaction times were in a range similar to the preceding experiments. Consequently, and to facilitate comparisons between experiments, the first 2 s of each trial were again excluded from further analysis for all but one of the of the extracted variables (time to onset of the first correct control episode). As mentioned in the discussion of Experiment 3, both the number of correct and incorrect control episodes have proven not to be related to the level of performance in terms of heading error. As a result both variables were dropped from subsequent analysis, and reaction time was now the only variable extracted from data sets that included the first 2 s of each trial.

Figure 25 indicates that, much like in Experiments 1 and 3, the different levels of absolute initial heading error ($|\theta_i|$) influenced performance for a large part of a trial's duration.

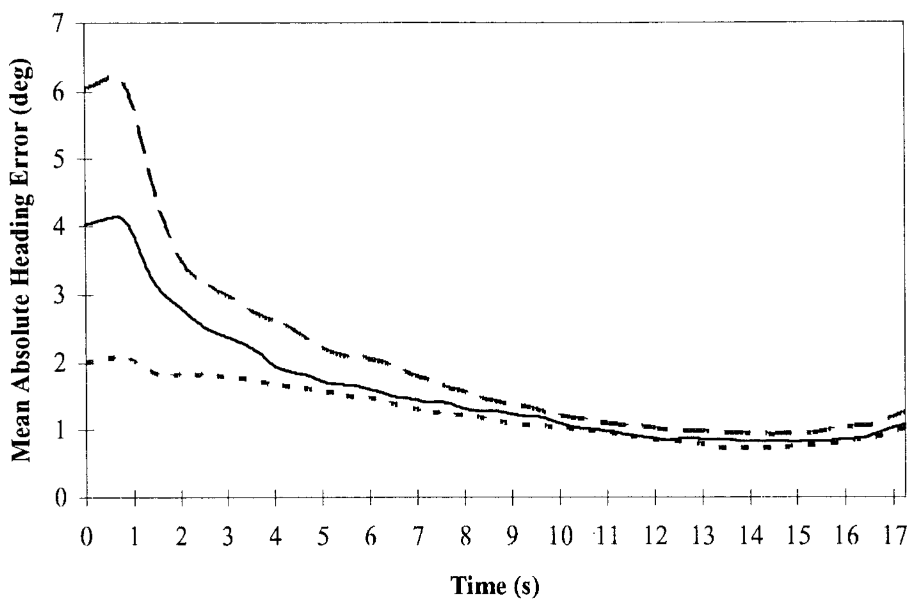


Figure 25. Mean absolute heading error (mean $|\theta|$) as a function of time and the three levels of absolute initial heading error ($|\theta_i|$). Excludes data from NDM environment. Global optical flow velocity = 1.2 h/s (each point on a curve represents the average of 264 observations).

(2) Analysis of the Extracted Variables

(a) Analysis 1. With exception of the number of both correct and incorrect control episodes, the same variables as in the preceding experiments were extracted from the raw data. A one-way MANOVA of all dependent variables of the form θ_i (left or right) yielded a nonsignificant main effect (Pillais $F(7, 916) = .4519$, ns). Consequently, the data were collapsed over left and right θ_i for all subsequent analyses.

To evaluate whether the data could be collapsed across the two SMP environments, a further one-way MANOVA of all dependent variables of the form Environment (SMPF or SMPB) was conducted. The result indicated that there was an overall performance difference (Pillais $F(7, 256) = 4.0461$, $p<.001$). The univariate tests indicated that this difference was mainly due to shorter reaction times ($F(1, 262) = 7.5508$, $p<.01$; $M_{SMPF} = 2.61$ s, $M_{SMPB} = 3.35$ s) and to smaller performance variability at the initiation of control episodes in the SMPF environment ($F(1, 262) = 5.1093$, $p<.05$; $M_{SMPF} = 1.28$ deg, $M_{SMPB} = 1.59$ deg). A very similar pattern of results was obtained by testing for performance differences between the three DMP environments: A one-way MANOVA of the form Environment (DMPF, DMPM, or DMPB) also revealed a significant multivariate effect (Pillais $F(14, 776) = 4.3439$, $p<.001$). The univariate tests again indicated that this difference was mainly due to differences in reaction time ($F(2, 393) = 20.5431$, $p<.01$; $M_{DMPF} = 1.42$ s, $M_{DMPM} = 1.91$ s, $M_{DMPB} = 2.51$ s) and to mean variability at the initiation of control episodes ($F(2, 393) = 3.8404$, $p<.05$; $M_{DMPF} = 1$ deg, $M_{DMPM} = 1.16$ deg, $M_{DMPB} = 1.32$ deg)[29]. As a consequence, the data could neither be collapsed across the two SMP environments nor across the three DMP environments. In the remainder of this section, the analysis proceeds in the same format as for the preceding experiments. In the subsequent section a closer look will be taken at the underlying causes for the observed differences.

A MANCOVA of all dependent variables of the form Environment (NDM, SMPF, SMPB, DMPF, DMPM, DMPB, or Random) x Visual Field (10.3 or 30.8 deg), with $|\theta_i|$ and trial number as covariates was conducted. This yielded a significant multivariate within-cells regression effect (Pillais $F(14, 1806) = 10.0482$,

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[29] A Bonferroni test of multiple comparisons showed that while for the measure of variability only the difference between the DMPF and the DMPM environment was statistically significant, reaction time differences between all pairs of the three environments reached that level (df = 393, $p<.01$),

$p<.001$) indicating that performance was influenced by the covariates. The univariate tests showed that all dependent variables except for the mean standard deviation of mean heading error at the conclusion of control episodes were strongly affected (Appendix K, Table K-1). The associated regression coefficients (Table 16, columns 1 and 2) indicate that performance deteriorated with increasing initial heading error, and improved somewhat with practice. The fact that the practice effect, although in the expected direction, lacks statistical significance is explained by the inclusion of the NDM environment in the analysis. Performance in this environment was generally very poor, with a mean heading error at the conclusion of control episodes of 12.95 deg. Since, consequently, accurate goal directed self-motion did not seem to be possible in this environment, a failure to achieve consistent improvement with practice seems plausible. To test for this possibility, the above analysis was repeated after excluding data from the NDM environment. Again, a significant multivariate within-cells regression effect (Pillais $F(14, 1546) = 24.9515$, $p<.001$) indicates the influence of the covariates. This time all dependent variables were affected (Appendix K, Table K-2), and the associated regression coefficients support the preceding argument (Table 16, columns 3 and 4), i.e., while the size of the practice effect remained the same, the reduced variability across cells favourably affected both F-ratios and regression coefficients. Note also, that the sizes of the coefficients associated with $|\theta_i|$ were now mostly smaller and consequently more in line with results from the previous experiments. Figure 26 serves to further illustrate the inability of subjects to successfully control the direction of simulated self-motion in the NDM environment. Consequently, the data from that environment were excluded from further statistical analysis in this section.

Table 16. Regression coefficients for the dependent variables associated with each of the covariates.

Dependent variables	Covariates			
	Full data set		Excl. data from NDM environment	
	$\|\theta_i\|$	Trial	$\|\theta_i\|$	Trial
Mean $\|\theta\|$ (deg)	.336***	-.005	.107***	-.007***
Mean $\|\theta\|$ during zero-ctrl	.301***	-.006*	.095***	-.008***
Mean $\|\theta\|$ at ctrl conclusion	.282**	-.007	.069**	-.006***
.....Associated SD	.103	-.002	.063**	-.002
Mean $\|\theta\|$ at ctrl initiation	.492***	-.006	.131***	-.008***
.....Associated SD	.122*	-.009*	.095***	-.005***
Reaction time (s)	-.260***	-.003	-.362***	-.006**

Note. θ_i = absolute initial heading error.

* p<.05, ** p<.01, *** p<.001

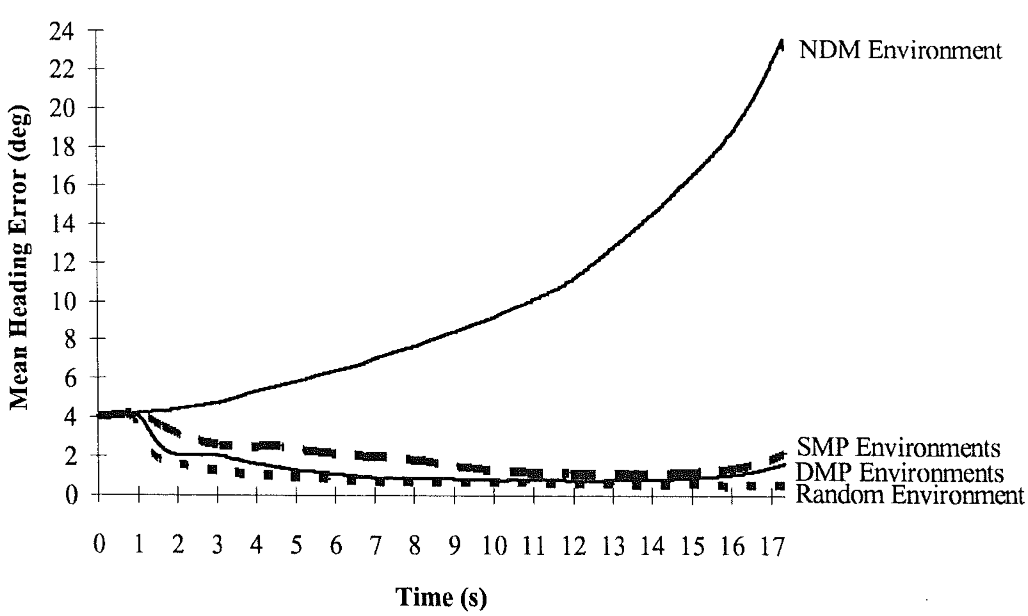

Figure 26. Mean absolute heading error (mean $|\theta|$) as a function of time and the four environments. Global optical flow velocity = 1.2 h/s.

To further analyse the effect of the different levels of initial heading error on performance and to see whether the pattern of results was similar to that obtained in Experiment 3, all truncated data sets were divided into four quarters. As in Experiment 3, only mean absolute heading error was extracted from the quartered data sets. An ANCOVA of the form Quarter (1st, 2nd, 3rd, or 4th) x $|\theta_i|$ (2, 4, or 6 deg) x Environment (SMPF, SMPB, DMPF, DMPM, DMPB, or Random) x Visual Field (10.3 or 30.8 deg) with trial as a covariate yielded significant effects for Quarter ($F(3, 3023) = 132.10$, $p<.001$), for the Quarter by $|\theta_i|$ interaction ($F(6, 3023) = 5.10$, $p<.001$), and for the Quarter by Environment interaction ($F(15, 3023) = 15.64$, $p<.001$). Note that as in Experiment 3, only effects involving Quarter are reported here, since the remaining effects are more appropriately dealt with by the main MANCOVA. The model explained 29.2 % of the total sum of squares (R^2), or 25.9 % of the variation in heading error (adjusted R^2). The Quarter by Environment interaction plotted in Figure 27 shows that, although generally performance improved with each quarter, there was an uncharacteristic deterioration of performance in two of the environments (SMPF and DMPF). On reflection, it became apparent that this deterioration occurred in the only two environments where no objects were placed behind the target. As a consequence, during the last part of the fourth quarter, the simulation was one of approaching a single fronto-parallel row of objects, much like in the NDM environment. The deterioration in the extracted performance measure reflects the fact that this kind of environment does not support accurate goal directed self-motion, as demonstrated in Figure 26. In addition, the interaction shows that performance differences between the environments decreased somewhat as the target was approached[30].

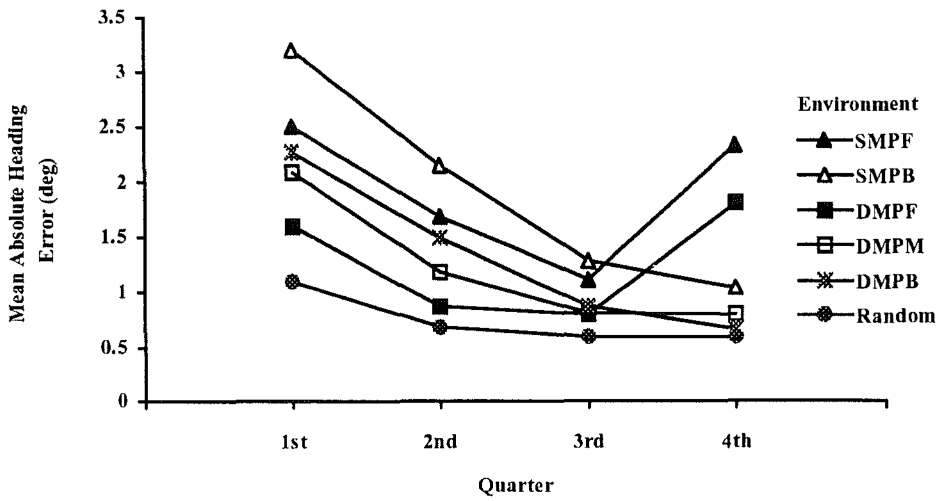

Figure 27. Mean absolute heading error (mean |θ|) as a function of trial duration and environment (excluding the NDM environment).

[30] The results of a Bonferroni multiple comparison are reasonably complex, but support this interpretation (df = 3023, p<.05): The performance decrease from the first to the second quarter is statistically significant for all environments, from the second to the third for all environments with exception of the DMPF and Random environments. From the third to the fourth quarter only the performance deterioration in the SMPF and DMPF environments is significant. Within the first quarter, the only non-significant differences were between the DMPB and two other environments (DMPM and SMPF). Within the second quarter, the only non-significant differences were between the Random and the DMPF environments, and between the DMPB and SMPF environments. In the third quarter the following means were not significantly different: Random and (DMPF, DMPM), DMPF and (DMPM, DMP), DMPM and DMPB, DMPB and SMPF, and SMPF and SMPB. In the fourth quarter, only the means for the DMPF and SMPF environments were significantly different from all other means. Note that for each multiple comparison only the highest p-value that is smaller than 0.05 is reported.

The interaction between Quarter and $|\theta_i|$ is represented in Figure 28. Again, it is apparent that performance generally improved over the four quarters. As suggested above, performance deterioration in environments SMPF and DMPF was responsible for the slight trend towards deterioration in the fourth quarter. Unlike in Experiment 3, the large performance differences in the first quarter were largely compensated for by the third. Nevertheless, the effect of the initial conditions in terms of heading error tended to influence performance throughout at least the first half of a trial's duration[31].

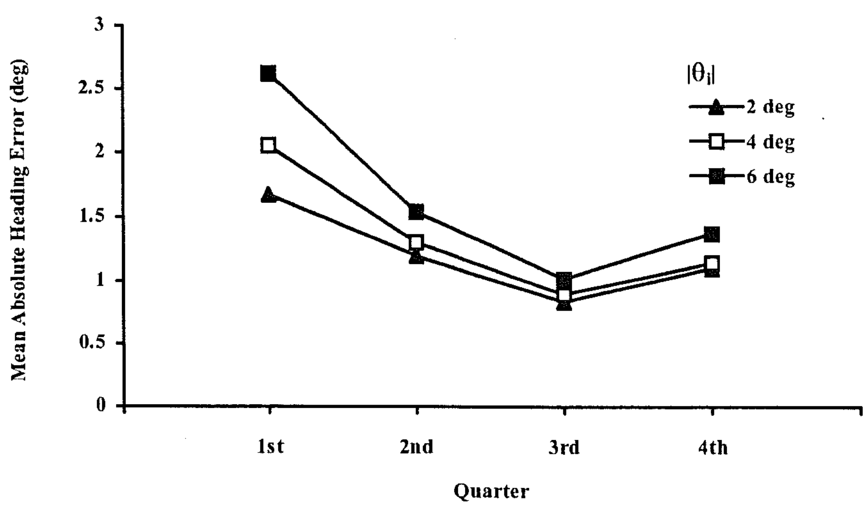

Figure 28. Mean absolute heading error (mean $|\theta|$) as a function of trial duration and absolute initial heading error ($|\theta_i|$).

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[31] A Bonferroni test of multiple comparisons showed all differences to be significant ($df = 3023$, $p<.05$), with the exception of the difference between initial heading errors of 2 and 4 deg in the second, third, and fourth quarters, and between 4 and 6 deg in the third quarter.

The main MANCOVA also detected a significant multivariate main effect for Environment (Pillais $F(35, 3880) = 8.5504$, $p<.001$). The univariate tests showed that all dependent variables were strongly affected (for details see Appendix K, Table K-3). Univariately, this effect expressed itself in two distinct patterns. (a) As far as the reaction time data were concerned (Figure 29), all differences with the exception of the one between the SMPF and the DMPB environment were significant (Bonferroni test of multiple comparisons, df = 778, $p<.05$). Apart from the fact that performance in the DMP environments was generally better than in the SMP environments, it is particularly interesting to note that there is a near linear performance deterioration from DMPF to DMPM and DMPB. (b) All performance measures based on heading error showed the pattern represented by mean absolute heading error during zero-control episodes as shown in Figure 30. Here, all differences with exception of the ones between the two SMP environments and the three DMP environments were significant (Bonferroni test of multiple comparisons, df = 778, $p<.05$). Finally, the MANCOVA failed to detect a significant difference between the conditions with the full and partial visual field (Pillais $F(7, 772) = .8346$, ns).

A Second Pass. In light of the finding that performance deteriorated in two of the environments (SMPF and DMPF) during the fourth quarter, all data sets were terminated at the point where the relevant type of information was still just available. In other words, data sets were truncated at the point where the simulated egolocus passed the point that was 4 eyeheights away from the target, i.e., at the point where the first row of objects passed from view in environment DMPF.

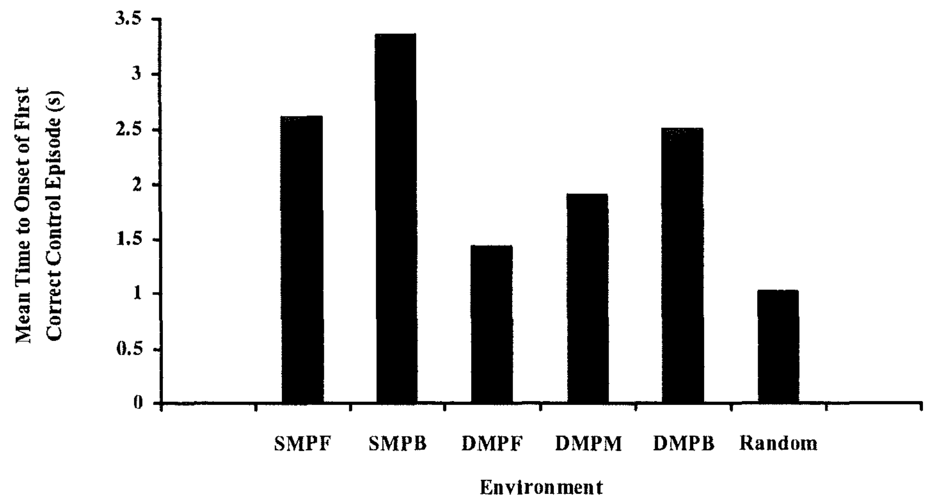

Figure 29. Mean time to onset of first correct control episode as a function of environment (excluding NDM environment).

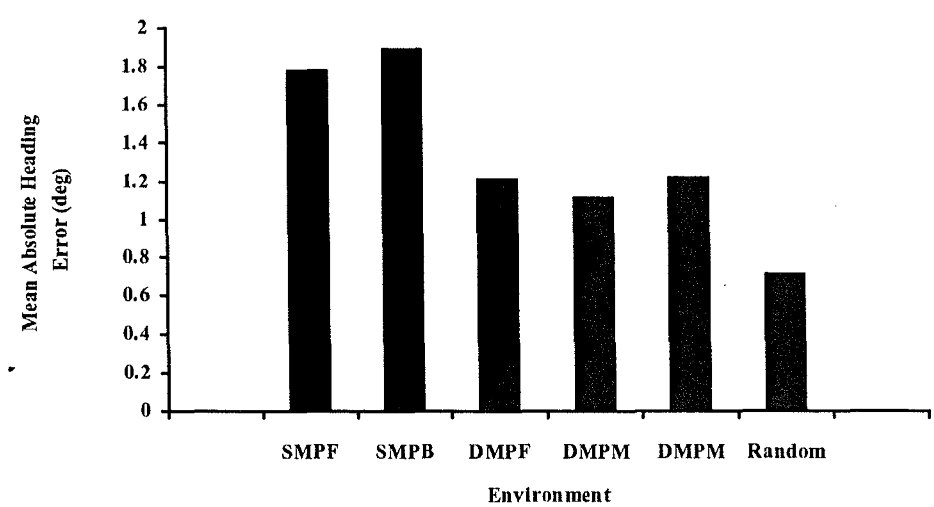

Figure 30. Mean absolute heading error (mean |θ|) during zero-control episodes as a function of environment (excluding NDM environment).

To check that the shortened data sets showed the same effects, the above analysis was repeated. Only the results that are at variance with those already reported are mentioned here: (a) There were now significant performance differences between the two SMP environments and among the three DMP environments on most of the measured variables (for details see Appendix K, Table K-4 and Table K-5). Performance on all dependent variables, with exception of mean standard deviation at the conclusion of control episodes, improved significantly from environment SMPB to SMPF, and from DMPB over DMPM to DMPF (Bonferroni test of multiple comparisons, df = 368, p<.05). (b) The distribution of the means associated with the still significant main effect for Environment (Pillais *F*(35, 3690) =6.45, p.<001) was changed: Since the performance differential between the environments during the last quarter was now effectively removed from the means, the pattern for all performance variables was now much closer to the pattern represented in Figure 29. To demonstrate, the values for mean heading error during zero-control episodes from the previous analysis (Figure 30) are plotted together with the new means (Figure 31). Unlike in the previous analysis, tests showed significant differences between all pairs of means, with the exception of differences between SMPF and DMPB, and between DMPM and DMPB. Note the much clearer increase from SMPF to SMPB, and from DMPF over DMPM to DMPB. This pattern of results was characteristic for all dependent variables (Bonferroni multiple test of comparisons, df = 740, p<.05).

The majority of the individual results for mean |θ| at the conclusion of control episodes were found to mirror the group effects. All subjects performed better in the SMPB than in the NDM environment, and in the DMPF than in the Random

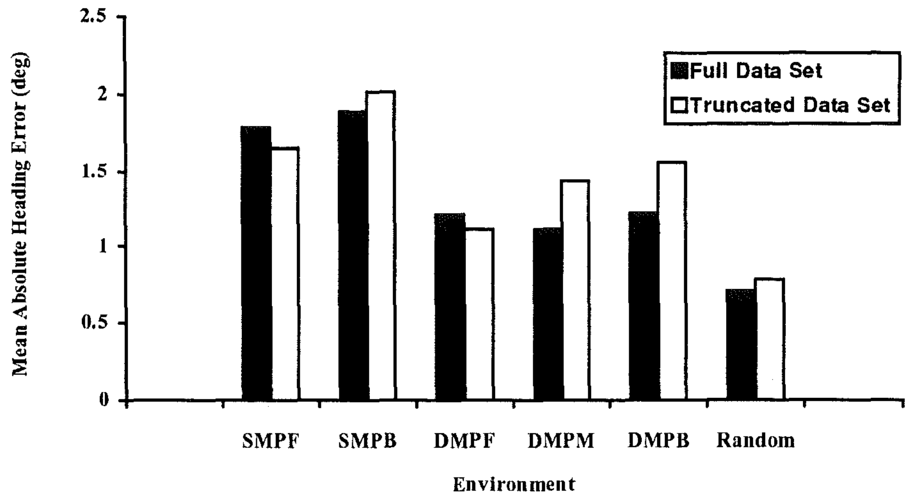

Figure 31. Mean absolute heading error (mean |θ|) during zero-control episodes as a function of environment (excluding NDM environment). The black bars are calculated from data sets terminated 90 samples before |θ| reached 90 deg, the white bars are from data sets truncated at a simulated distance of 4 h from the target.

environment. Similarly, the performance of eight of the eleven subjects was better in the SMPF than the SMPB environment, six were better in the SMPB than the DMPF environment, nine were better in the DMPF than the DMPM environment, and seven were better in the DMPM than the DMPB environment[32].

(b) Analysis 2: The Separation Ratio and an Alternative Design. At first glance then, it seems that differential motion parallax may indeed be a functional type of information for the perception and control of heading. Nevertheless, the question as to why there were performance differences between the two SMP environments and the three DMP environments has yet to be addressed. It is suggested that the

[32] Any individual effects not mentioned here were either near 0 or slightly in the opposite direction of the group effect.

introduction of σ (Equation **3**, p. 97) may go some way to providing the answer. Initially, it was intended to be introduced into the MANCOVA as a covariate. To do so, σ was calculated for each sample at the moving point of observation, where $d_{A\text{-}NO}$ was the shortest distance between the moving point of observation and the row of objects closest to it, and $d_{A\text{-}FO}$ was the shortest distance between the moving point of observation and the row of objects farthest away from it. The final values that were to be used for the analysis were simply the mean separation ratios for each trial[33]. However, since the resulting values appeared to be nearly linear combinations of the independent variables (Table 17), the intended type of analysis was not appropriate.

At this point it seemed advisable to reconsider the types of information contained in the simulated environments. (a) The NDM environment had been shown to contain no functional information (NFI) for the perception and control of heading. (b) Both SMP environments contained simple motion parallax information, the magnitude of which is partly defined by σ. (c) All three DMP environments contained DMP *as well as* SMP information, both partly defined by σ. This is the observation that pointed towards an alternative type of analysis. The Random environment also contained those same types of information as well as other, unspecified information (UI)[34].

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[33] It was impractical to calculate the separation ratio in its present simple form continuously for the Random environment, so a number of trials were sampled and the ratio was calculated at five fixed intervals. Most values arrived at in this manner were between 0.9 and 1.0. Consequently, the mean separation ratio was estimated to be at a conservative 0.9 for all trials in the Random environment.

[34] Two examples include locomotor flow line information (Lee & Lishman, 1977) and a global optical flow that is evenly distributed throughout the visual field (Perrone & Stone, 1994).

Table 17. Mean separation ratio (σ) for all environments.

Environment	Mean σ	Standard Deviation
NDM	0	0
SMPF	.3847	.0002
SMPB	.2969	.0002
DMPF	.5856	.0002
DMPM	.5298	.0002
DMPB	.4437	.0002
Random	.9	0

Consequently, the independent variables were reorganised into three groups. The first group contained data from the NDM environment, the second contained data from the two SMP environments, and the third contained data from the three DMP environments. In addition, for some of the following analyses the data from the Random environment were included in the third group. The resulting general design was of the form Type of Information [NFI, SMP, or DMP(+UI)] by Visual Field (10.3 or 30.8 deg) with mean σ, trial number, and $|\theta_i|$ as covariates.

Analysis 2a: Recapitulation. To confirm that the alternative type of analysis yields comparable results, it was initially performed without mean σ, and excluding the data from the NDM environment, i.e., the analysis was of the form Type of Information (SMP, or DMP+UI) by Visual Field (10.3 or 30.8 deg) with trial number and initial $|\theta_i|$ as covariates. This should mirror results obtained by the analysis of the previous section. Results of the within-cells regression remained identical to the ones obtained in the previous analysis (Pillais $F(14, 1486) = 24.9185$, $p<.001$). In addition, neither the Visual Field effect nor the interaction between Visual Field and Type of Information was significant (Pillais $F(7, 742) = 1.1042$, ns, and Pillais $F(7, 742) = 1.5782$, ns, respectively). Finally, there was again a strong multivariate main effect for Type of Information (Pillais $F(7, 742) = 18.8299$, $p<.001$). Since for the present analysis the data from the DMP and Random environments were pooled, this reconfirms that performance in environments that made SMP, DMP, and other unspecified information available was better than in environments that made only SMP information available.

To assess whether performance in the three DMP environments was better than in the two SMP environments, a partial MANCOVA of the same form, but excluding data from the Random environment was performed. Much like before, the only significant effects that were obtained were the within-cells regression effect (Pillais $F(14, 1242) = 24.6541$, $p<.001$) and the main effect for Type of Information (Pillais $F(7, 620) = 10.9502$, $p<.001$). The regression coefficients associated with the analysis of the covariates showed the by now familiar pattern, i.e., performance improved with practice, and deteriorated with increasing $|\theta_i|$. Similarly, the univariate tests of the main effect indicated that all dependent variables (with the exception of mean standard deviation at the conclusion of control episodes) were equally strongly affected (for details see Appendix K, Table K-6). The corresponding means indicated that performance was better in the DMP than the SMP environments.

In sum, the alternative type of analysis yielded results that were identical to the ones obtained in Analysis 1. It has now arrived at the point where the previous approach (Analysis 1) left off. In contrast however, it is now possible to introduce mean σ as an additional covariate. If the ratio has psychological reality, the previously observed performance differences between the environments should disappear, or at least become considerably smaller as they are adjusted for the effect of the ratio. In other words, if performance varies with mean σ, its introduction as a covariate will remove its effect from the means, thereby reducing the observed performance differences.

Analysis 2b: NFI versus SMP versus DMP+UI. Initially, the data from all environments were included to test for the possibility that the ratio is able to account for performance differences between all tested environments. This analysis yielded a significant within-cells regression effect (Pillais $F(21, 2442) = 12.4993$, $p<.001$). Univariate tests showed that all dependent variables were strongly affected (for details see Appendix K, Table K-7). The regression coefficients associated with mean σ (Table 18) demonstrate that performance improved along all measured dimensions as the ratio increased. To prevent repetition, regression results for trial number and $|\theta_i|$ will not be repeated here.

The analysis also yielded significant multivariate main effects for Visual Field (Pillais $F(7, 812) = 5.3507$, $p<.001$), for Type of Information (Pillais $F(14, 1626) = 25.9338$, $p<.001$), as well as for the interaction term (Pillais $F(14, 1626) = 3.5372$, $p<.001$). The univariate tests of the interaction effect show that only one of the dependent variables (mean heading error at the conclusion of control episodes) was affected ($F(2, 818) = 3.482$, $p<.05$). The same variable was responsible for the multivariate main effect of Visual Field ($F(1, 818) = 7.898$, $p<.01$). Examination of Figure 32 shows that performance was still generally poorest in the NDM

Table 18. Regression coefficients for the dependent variables associated with the mean separation ratio (σ).

Dependent variables	Regression Coefficients
Mean $\|\theta\|$ (deg)	-1.798***
Mean $\|\theta\|$ during zero-control	-1.715**
Mean $\|\theta\|$ at control conclusion	-1.755**
.....Associated SD	-1.232***
Mean $\|\theta\|$ at control initiation	-2.557***
.....Associated SD	-1.610***
Reaction time (s)	-3.189***

Note. Analysis includes data from the NDM and Random environments.

** p<.01, *** p<.001

environment (Bonferroni test of multiple comparison, df = 818, p<.001). In addition, the presence of the peripheral visual field resulted in some improvement in that environment (p<.001). This result is understandable in light of the consideration that the amount of differential optical motion within any one row of objects increases with the size of the visual field. Since it was the only type of specific information available in the NDM environment, it was used to its fullest potential resulting in a small but measurable performance improvement in the NDM environment with the wide field of view. No further significant differences were detected (df = 818, ns). Apparently, the SMP and DMP+UI environments provided enough functionally rich information in the central visual field that this additional (and on its own inadequate) type of information could be, and was, disregarded. In either case, the fact that only one of

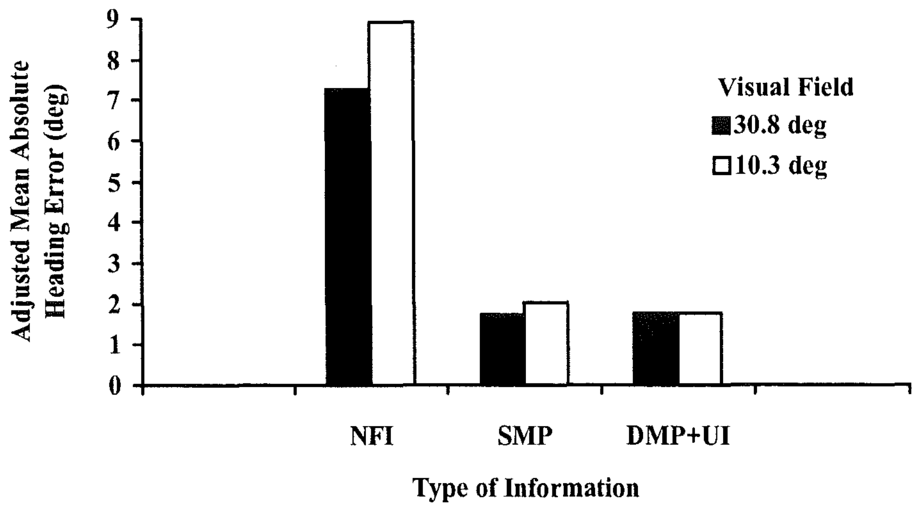

Figure 32. Mean absolute heading error (mean |θ|) at the conclusion of control episodes adjusted for the covariates as a function of Type of Information and Visual Field (NFI = no functional information, SMP = simple motion parallax, DMP+UI = differential motion parallax and unspecified information).

the dependent variables was affected, and that this was the most sensitive of the measures related to heading error[35] emphasizes both the weakness of the functionality of information made available by a single fronto-parallel row of objects, and the small degree of importance of peripheral field of view information for the task of target approach. Most important for the present purposes however, was *the failure to detect any significant differences between the SMP and DMP+UI environments.*

Keeping this in mind it is advisable to examine the main effect for Type of Information prior to further interpretation: The univariate test results show that all

[35] Recall that this measure takes only those values of absolute heading error into account that conclude a control action, i.e., when the controller is satisfied that he is heading where he intends to head (disregarding system limitations).

dependent variables were strongly affected ($p<.001$, for further details see Appendix K, Table K-8). All of the dependent variables showed the same pattern of adjusted means: Performance improved from the NDM (NFI) to the SMP environments (Bonferroni test of multiple comparisons, $df = 818$, $p<.001$), but remained constant from the SMP to the DMP+UI environments.

Taken together these results indicate that variation in σ was able to account for most of the observed variation in performance. Nevertheless, as the still considerable differences in adjusted performance measures between the NDM and SMP environments show, it was not able to accommodate the extreme case of approach to a fronto-parallel plane, where the amount of differential optical motion is virtually nil. Although in this case σ takes a value of 0, it still overestimates performance. At least some separation in depth between optical elements seems essential (as will be shown later, a log transformation may be effective here). Nevertheless, perhaps the most striking result is the complete lack of a difference in adjusted performance measures between the SMP environments and the DMP+UI environments. This stands in sharp contrast to the previous analysis, where performance in the Random environment was consistently better than in all others, and performance in the DMP environments was consistently better than in the SMP environments.

Analysis 2c: SMP versus DMP. It was now of interest to determine whether variations in σ could account equally well for the performance differences between the two SMP environments on one hand, and the three DMP environments on the other (excluding data from the NDM and Random environments). To answer this question, a second partial MANCOVA of the form Type of Information (SMP or

DMP) by Visual Field (10.3 or 30.8 deg) with trial number, $|\theta_i|$, and mean σ as covariates was performed. This yielded a significant multivariate within-cells regression effect (Pillais $F(21, 1863) = 18.2921$, $p<.001$). All dependent variables were strongly affected (for details see Appendix K, Table K-9). The regression coefficients associated with mean σ are represented in Table 19. Again, performance was found to improve with increasing mean σ.

Table 19. Regression coefficients for the dependent variables associated with the mean separation ratio (σ).

Dependent variables	Regression Coefficient
Mean $\|\theta\|$ (deg)	-4.073***
Mean $\|\theta\|$ during zero-control	-4.199***
Mean $\|\theta\|$ at control conclusion	-3.100***
.....Associated SD	-1.832*
Mean $\|\theta\|$ at control initiation	-5.791***
.....Associated SD	-3.073***
Reaction time (s)	-8.058***

Note. The analysis excludes data from the NDM and Random environments.

* p<.05, *** p<.001

The analysis also detected a significant multivariate main effect for Type of Information (Pillais $F(7, 619) = 2.173$, $p<.05$). Results of the univariate tests show that this effect was mainly due to differences in mean standard deviation of heading error at the initiation of control episodes ($F(1, 625) = 3.8923$, $p<.05$, for further details see Appendix K, Table K-10). The relevant adjusted means indicated that performance was *more* variable in the DMP environments (adjusted M_{SMP} = 1.0 deg, adjusted M_{DMP} = 1.32 deg). No further significant differences were detected. This indicates that variations in σ can account for the observed performance differences between environments containing simple and differential motion parallax.

The usefulness of the type of analysis employed in Analyses 2a through 2c is now exhausted. Nevertheless, one question still remains unanswered. Although it has been shown that variations in σ can account for performance differences between environments containing a variety of types of information, it is not yet clear to what extent it can do this. The following section attempts to formally answer that question.

(c) Analysis 3: Percent of Variance and Multiple Regression. The concept of 'proportion of variance accounted for' does not carry over very well from a univariate to a multivariate approach to data analysis. Nevertheless, for the sake of completeness, a multivariate alternative is given by the formula

$$R_M^2 = \sum_{n}^{1} (1-\lambda), \tag{4}$$

where R_M^2 is the multivariate approximation of the univariate R^2, n is the number of effects in an analysis (including the within-cells regression effect), and λ is the value of Wilks λ for each effect. The R_M^2 values for Analyses 1 to 2c are represented in Table 20. Keep in mind however, that these values are not strictly equivalent to the univariate R^2. For example, in certain circumstances R_M^2 can take values that are greater than 1. Nevertheless, one can see that as mean σ is introduced, the R_M^2 values for the regression increase, while those associated with the Type of

Information effect decrease. Of particular importance is the fourth row of the table (Analysis 2c) since it relates exclusively to the comparison between simple and differential motion parallax. Note also that the inclusion of the Random environment into the design of Analysis 2b is chiefly responsible for the relatively large R_M^2 value of the corresponding Type of Information effect.

Table 20. Multivariate approximations of 'proportion of variance accounted for' (R_M^2) for each effect of Analysis 1 (data from NDM environment excluded, second pass), 2a (data from NDM environment excluded, alternative design), 2b (full set of data, alternative design), and 2c (data from NDM and Random environments excluded, alternative design).

			Main Effects		
Analysis	**Within-cells Regression**	**Interaction Effect**	**Peripheral Field of View**	**Type of Information***	**Total**
1	.372	.045	.010	.278	.705
2a	.371	.015	.010	.173	.569
2b**	.275	.059	.044	.362	.740
2c**	.469	.017	.010	.023	.519

* For Analysis 1 this is the Environment effect. ** Analysis includes mean σ.

A further point requires clarification: The tests associated with the within-cells regression of MANCOVA are restricted to calculations within the defined experimental cells of each analysis, and consequently fail to convey an accurate idea of the total strength of the relationship (Figure 33).

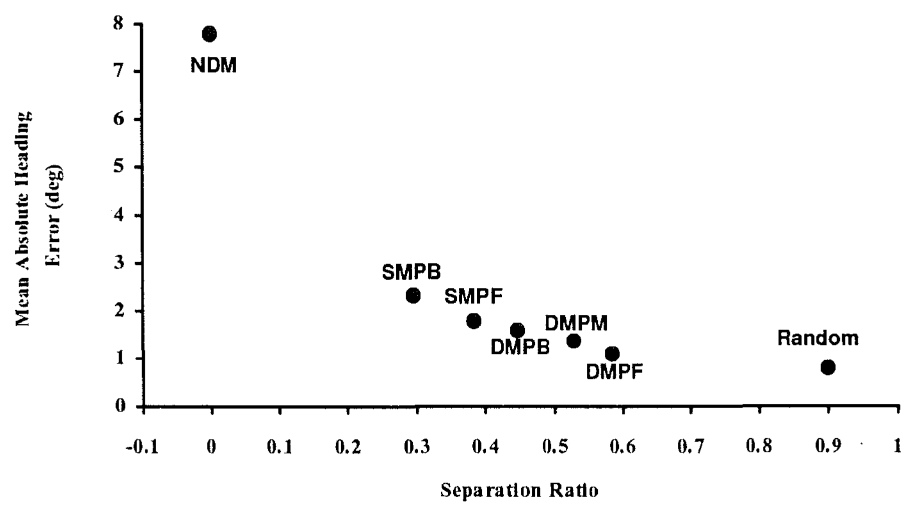

Figure 33. Scatterplot of mean absolute heading error (mean $|\theta|$) during zero-control episodes as a function of the mean separation ratio (σ) for each environment.

A reasonable alternative that has no such restrictions is provided by multiple regression. Although it is a univariate technique, and consequently does not allow concurrent evaluation of effects on the basis of more than one dependent variable, it will increase understanding of the relationship between performance and σ. A dummy variable called FIELD was coded 0 for those trials in which the visual field spanned 10.3 deg, and 1 for all those in which it spanned 30.8 deg. In addition, $|\theta_i|$, trial number, and mean σ were introduced into the analysis as ABSFRO, TRIAL, and SRATIO respectively. The resulting general equation was

$$\text{PREDvar} = \alpha + \beta 1(\text{ABSFRO}) + \beta 2(\text{TRIAL}) + \beta 3(\text{FIELD}) + \beta 4(\text{SRATIOx}) + \varepsilon, \quad (5)$$

where PREDvar is a predicted dependent variable, α is the corresponding population constant, $\beta 1$ to $\beta 4$ are the population regression coefficients associated with each of the predictor variables, and ε is the experimental error. Since it was of interest to determine how much of the variation in the data mean σ (SRATIO) was able to explain *after* all other known factors were accounted for, it was only introduced into the regression after $|\theta_i|$ (ABSFRO), trial number (TRIAL), and Visual Field (FIELD) had been entered. In addition, SRATIO was only entered if it accounted for a significantly large additional proportion of the variance. The regression coefficients for each dependent variable along with the appropriate F ratios and R^2 values are represented in Table 21. Second, acting on the finding that mean σ overpredicted performance in the NDM environment, the procedure was repeated for the log-transformed dependent variables (LOG-PREDvar). The corresponding R^2 values [R^2(log)] for these calculations are included in the table for comparison purposes.

Overall, the separation ratio was able to account for 30 to 40 % of the variation in performance for all dependent measures related directly to heading error and reaction time. It was somewhat poorer at accounting for variability of performance (around 10 % of the additional variation in the two measures constructed from the standard deviations). Note that all R^2(log)-values bar one show an improvement of between 2 and 8 percentage points.

Table 21. Regression coefficients, F-ratios, and additional proportion of variance accounted for [R^2, R^2(log)] by the introduction of the mean separation ratio (SRATIO) into Equation 5.

Dependent Variable	CONSTANT	B1 ABSFRO	B2 TRIAL	B3 FIELD	B4 SRATIO	F	R^2	R^2(log)
Mean |θ| (deg)	4.65***	.32***	-.01	.09	-7.29***	157.94***	.37	.41
Mean |θ| during zero-ctrl	4.64***	.32*	-.01	.05	-7.29***	144.49***	.35	.39
Mean |θ| at ctrl conclusion	4.88***	.15*	.00	.22	-6.96***	71.59***	.24	.26
.....Associated SD	1.63***	.08**	.00	.03	-1.79***	23.11***	.09	.17
Mean |θ| at ctrl initiation	5.28***	.30***	-.01*	.07	-6.82***	100.59***	.29	.27
.....Associated SD	2.41***	.04	.00	.05	-2.88***	26.12***	.11	.13
Reaction time (s)	9.03***	-.29***	-.01	-.01	-9.76***	165.24***	.41	.49

Note. F-ratios correspond to entire equation. B1 to B4 are the estimates of the population regression coefficients β1 to β4.

* $p<.05$, ** $p<.01$, *** $p<.001$

III. DISCUSSION

(1) Effect of Initial Conditions

The data from Experiment 3 indicated that the participants had not been able to compensate for the difference between absolute initial heading errors ($|\theta_i|$) of 2 and 4 deg in the time available to them. This result had been interpreted to indicate that the level of $|\theta_i|$ may have helped determine a visual criterion for acceptable performance that was then adhered to until the target was reached (perceptual anchoring). In contrast, the results of Experiment 4 showed that, although differing levels of θ_i again influenced performance for a large part of a trial's duration, these differences were largely compensated for by the third quarter. An explanation in terms of an *ongoing* perceptual constraint is not supported by the data. Had initial heading error acted as such, the performance difference between the conditions should have remained constant throughout the entire event. However, an explanation purely in terms of limitations of the action part of the system (i.e., participants could see that they went wrong, but were not able to correct) is similarly implausible. While results of Experiment 4 have indicated that there were still performance differences after 10 s, some of the trials in Experiment 3 (those with high global optical flow velocity) only lasted a total of 10 s; yet performance in these trials was better than in trials that lasted longer. Taken together, this pattern of results suggests that, rather than acting as an ongoing and purely perceptual constraint, θ_i temporarily constrained the perception-action system as a whole. On one hand, some time was without a doubt spent trying to achieve the intended flow of visual information, i.e., negating an error of 6 deg would necessarily take a little longer than one of only 2 deg (Fitts, 1954). On the other hand, if this had been the only constraint, initial differences should have been compensated for much quicker than they actually were. As a result it is suggested that the initial level of heading error helped establish a temporary criterion against which improvement was evaluated. During the course of a trial, this criterion was gradually updated by more recent visual information, and differences in performance disappeared[36].

(2) Field of View

Results of Experiment 3 established that the addition of peripheral SMP information to an environment that carried splay and global optical flow information led to a small but measurable performance improvement on two of the dependent variables. In contrast, addition of DMP information in the peripheral field of view to a display that already contained peripheral SMP information did not lead to a significant performance improvement on any of the measured variables. In Experiment 4 the peripheral field of view was blocked out on half of the trials to determine whether motion parallax information presented peripherally influences the perception and control of heading (in the presence of information in the central field of view). Results indicated that it is of limited functionality. In addition, it is used only if information available in the central field of view is of insufficient functionality for the task at hand. From a limited resource point of view these findings make good sense. As long as centrally available information is sufficiently informative about a task, additional evaluation of peripheral information is of little advantage. It is only

[36] Note that this interpretation cannot account for the fact that the performance difference between the two lower levels of initial heading error in Experiment 3 persisted throughout the entire event.

when centrally available information does not carry enough functionality to complete a task successfully, that peripheral information is taken into account. Consequently, the use of wide-angle displays may be advantageous for the perception and control of heading during target approach, but only when there is a paucity of functional information in the central display or around a target.

(3) Motion Parallax, Target Drift, and σ

The results of Experiment 4 replicated those of Experiment 3 in so far as performance was found to improve as the distance between the simulated point of observation and the target decreased. To account for this finding, a ratio was introduced that indexes the difference in distance between the observer and two objects in approximately the same vertical sector of the optical array (Equation 3, p. 97). Because its value increases with increasing separation in distance between the two reference points and the observer, it was termed separation ratio (σ). If both reference objects are equidistant from the observer, σ takes the value of 0. As the objects are approached, σ increases exponentially, until reaching a value of 1 as the first object is passed.

The simulated environments were designed to allow an evaluation of the relative contribution of simple and differential motion parallax information to the perception and control of heading. In the NDM (no differential motion) environment, approach to a row of objects approximately perpendicular to the line of travel was simulated. Although both horizontal and vertical components of global optical flow were present, the fact that there was very little variation in depth meant that there was hardly any differential optical motion. As expected, performance in this environment was found to be inadequate to achieve goal directed self-motion. Nevertheless, the fact that performance was better than in the target-only condition of Experiment 2 indicates that some functional information was present in the simulation. Recall that the simulated viewing direction remained centered on the target throughout each trial. As the simulated point of observation came close to passing the target, the single row of objects came to be viewed from an angle much smaller than 90 deg. During this brief period differential optical motion became increasingly available to the actor, allowing 'last minute' adjustments to be made. Overall, however, the results are in line with findings from passive observation studies indicating that the error in making heading judgments during approach to a plane perpendicular to the axis of linear translation is too large to allow goal directed self-motion (Llewellyn, 1971; Johnston et al., 1973; Regan & Beverley, 1982). Consequently, it seems established beyond reasonable doubt that veridical perception of heading and its successful control require a degree of variation in depth. On the other hand, research by W. H. Warren et al. (1988) and, more recently, W. H. Warren and Saunders (1995) found reasonable accuracy of heading judgments for approach to a fronto-parallel plane constructed of random dots with and without the presence of independently moving objects. However, unlike in Experiments 2 through 4, their displays contained target drift as a functional type of information. Results from Experiments 1 and 2 indicate that a high degree of accuracy can be achieved by the acquisition of heading information conveyed by target drift. Although W. H. Warren et al. (1988) claimed to have eliminated target drift by presenting the *nominal target* on the horizon, and only after display motion had stopped, nothing prevented subjects from designating any of the optical discontinuities during the simulation as *effective target*. A target is simply whatever optical discontinuity the path of self-motion is evaluated against. In support of this interpretation is the fact that when the FRO was obscured by a moving (random dot) object in W. H. Warren and Saunders' (1995) study, judgment accuracy was impaired. This result is consistent with a loss in sensitivity when there is a lack of optical motion around the FRO, which is simply the absence of target drift. The finding that when the moving object obscuring the FRO was black, performance did not deteriorate, may have been due to the fact that such an object provides less optical 'confusion' than a random dot object that moves against a background that is also constructed of random dots.

Performance in the simple motion parallax (SMP) environments was found to be superior to that in the NDM environment, and performance in the differential motion parallax (DMP) environments was found to be superior to that in the SMP environments. The former finding indicates that the information conveyed by the relative optical motion of two discontinuities provides a high degree of functionality for the perception and active control of heading during simulated self-motion. The latter finding could be taken to indicate that DMP contains functional information over and above that provided by SMP. This interpretation would be consistent with Cutting (1986), Cutting et al. (1992), and, more recently, Vishton and Cutting (1995), and Cutting et al. (1995). However, the fact that performance also differed between the two SMP environments and among the three DMP environments led to the search for an alternative interpretation. As it turned out, σ, originally intended to account only for performance improvements with decreasing distance to the target, was also able to explain most of the performance differences between all of the simulated environments. Of particular interest in the present context was that, once the effect of σ was accounted for, performance did not seem to benefit from the addition of DMP information to environments that already contained SMP information. As a side issue, the ratio was found to overestimate performance in the NDM environment where the target was part of a single row of objects oriented at right angles to the movement path ($\sigma = 0$), and underestimate performance in the reasonably cluttered Random environment ($\sigma = 0.9$). In other words, when the ratio was near zero, small increases resulted in large performance improvements. When it was near one, the same increases resulted in small performance improvements. A log-transformation of the dependent variables improved the explanatory power of a multiple regression analysis, thereby supporting this interpretation.

The second issue that can now be addressed is whether σ, which was found to be a determinant of performance in Experiment 4 can also explain some of the variation in performance observed in Experiment 3. Recall again that in Experiment 3 performance in both SMP and DMP environments was a little better than in the NMP (no motion parallax) environment, and that it was best in the Random environment (for details see Table 12, p. 90). If, as argued earlier (see p. 111), all simulated environments of Experiment 3 contained some motion parallax information, and σ is a determinant of the degree of the functionality of that information, it should be possible to show that performance varied with σ. Given that information in the central field of view is of greater importance, the separation ratio (Equation 3, p. 97) for the NMP environment of Experiment 3 may be calculated by using the distance between observer and the target as $d_{A\text{-}NO}$, and the distance between observer and the last object in either of the two rows as $d_{A\text{-}FO}$[37]. In the SMP environment a second row

[37] For a detailed description of all environments refer to Appendix A.

of objects was introduced on either side of the target. The separation ratio for this environment may be calculated by again substituting the distance between observer and target for $d_{A\text{-}NO}$, and, since the newly introduced rows of objects extend further to the horizon, the distance between the observer and the last object in either one of these rows for $d_{A\text{-}FO}$. In the DMP environment, a third row of objects was introduced on either side of the target. However, since it was placed between the two already existing rows of objects on each side, and extended only a fraction of the distance of the outside row to the horizon, the separation ratio remained unaffected. The resulting values for σ are then 0.52, 0.77, 0.77, and 0.9 for the NMP, SMP, DMP, and Random environments, respectively. Two limiting factors need to be taken into account. (a) Note that the (initial) horizontal optical angular deviation between the two objects defining the ratio in the NMP environment was smaller than in the SMP and DMP environments (4.5 and 6.5 deg, respectively). Assuming that it becomes more difficult to acquire information from the differential flow of two discontinuities (e.g., Rieger and Lawton's (1985) difference vectors) as their optical separation increases (Gibson et al., 1955), it is suggested that this served to limit the performance improvement one would expect from an increase of σ from 0.52 to 0.77 on the basis of data from Experiment 4 (Figure 33). (b) The far object used in the calculation of the ratio for the SMP and DMP environments ($d_{A\text{-}FO}$) would frequently have been occluded by objects in the nearer rows, thus further diminishing the utility of the increased value of σ. In contrast, these two factors had the opposite effect for the comparison between the Random environment and the two motion parallax environments: The optical separation between the objects used in the calculation of σ was smaller in the Random environment than in the two motion parallax environments. In addition, occlusion of those optical discontinuities that provided the largest value for σ in the Random environment was rare. Consequently, the performance improvement should be larger than the one that could be expected from an increase of σ from 0.77 to 0.9.

The observed performance differences between the simulated environments are entirely consistent with this interpretation. Much like in Experiment 4, variations in σ can thus be used to satisfactorily explain the observed variations in performance. In sum, the introduction of the separation ratio has been able to account for almost all of the observed performance differences between the simulated environments. This suggests that, as long as there is some variation in depth between the optical discontinuities of the optical array, it is not important whether there are three discontinuities that allow the emergence of motion parallax information or only two. The deciding factor is the degree of separation between the optical elements that are attended to and their distance from the moving observer. This relationship seems captured to some degree by σ. Similarly, σ can account for evidence from a passive observation study that (unexpectedly) indicated increased accuracy of heading judgments during simulations of linear self-motion with decreasing distance to a simulated fixation object (Cutting et al., 1995)[38].

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[38] The investigators attributed this effect to the increased retinal velocities of nonfixated objects in both the foreground and background when an observer fixates a near object.

CHAPTER VI

GENERAL DISCUSSION AND CONCLUSIONS

The four experiments described here represent the first systematic investigation into the effects of relative optical motion on the perception and control of heading during target approach. In general, the observed range of heading thresholds confirms the utility of an active control paradigm for the study of heading perception. Performance in information rich environments was excellent (with heading errors of less than 1 deg) and compares favourably to the passive observation literature. Since the level of gain associated with the control device was not formally optimized, it is possible that there is a potential for even better performance. On the other hand, the poorest performance of more than 17 deg error (Experiment 2, Target-only environment) guarantees that the potential performance range is large enough to allow successful discrimination between experimental conditions without sacrificing ecological validity.

Analysis of the participants' performance in each of the four experiments yielded a plethora of interesting results. First a caveat for researchers who might wish to also adopt an active control paradigm for the study of self-motion perception. Unless visual feedback is given to subjects with regard to acceptable performance, they may evaluate their performance throughout a trial with reference to the conditions at the onset of the trial. This effect may or may not decrease as trial duration increases. Two solutions to this problem present itself. In most instances it will be possible to control for this effect. This was the approach taken in the present investigation. While this has the advantage of allowing assessment of the strength and duration of the effect, it may in many instances be more advantageous to eliminate this additional source of variation. In the present context this could be achieved in one of two ways. A preview period could be included, during which linear approach to the target is simulated, followed by a veering off to one side, after which the observer would take control[39]. Alternatively, a disturbance function could be used to perturb an otherwise linear approach to a target. The observers' corrective actions could then be used to evaluate perceptual performance. This approach has been successfully used to study the perception and control of speed of self-motion (e.g., Zaff & Owen, 1987), altitude (e.g., Owen et al., 1995), and heading (Laudeman & Johnson, 1993).

Another 'incidental' observation is worth mentioning here. Data from Experiment 1 revealed that during simulations that employ a fixed angle between the viewing direction and the direction of self-motion, there is a bias to perceive the aimpoint closer to the center of the display than it actually is. This 'aperture bias' is well documented in the passive observation literature (Cutting et al., 1992; Gibson, 1947; Johnston et al., 1973; Llewellyn, 1971; W. H. Warren, Mestre et al., 1991) and is now also demonstrated to be operational during the active control of simulated self-motion.

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[39] Of course this would also serve to decrease the usefulness of the reaction time measure.

I. RATE OF EXPANSION, TARGET ROTATION, AND TARGET DRIFT

Comparison of Experiments 1 and 2 allowed evaluation of three potentially functional types of information for the perception and control of heading: (a) The rate of expansion appeared to be a non-informative type of information for the perception and control of heading. (b) Similarly, the optical rotation of the target was shown to be of insufficient functionality with the relatively featureless objects used in the present study. A recent investigation into collision avoidance serves to support a suggestion made earlier that irregular, texture-rich objects might be expected to provide enough of this kind of information to make goal-directed self-motion possible. Cutting et al. (1995) simulated linear self-motion in the presence of a moving pedestrian in an otherwise clutterless environment. Results indicated that the information contained in the apparent rotation of the simulated pedestrian was of sufficient functionality to allow above chance accuracy of heading judgments. (c) Finally, the comparison indicated that target drift, i.e., the optical movement of a target with respect to the frame of a window into an environment, contains sufficient functionality for the perception and control of heading during self-motion events. A case was also made for its functionality during self-motion events where that window is not present. In this instance target drift is perceived with respect to the self. Additional evidence comes from a study by Beall and Loomis (1995b) who found that information conveyed by target drift (defined as motion parallax) enabled subjects to keep to a straight path in the presence of laterally perturbing forces during simulated flight. Similarly, an investigation of eye movements during driving found that people spend a considerable amount of time looking at the 'tangent point' on the inside of a curve (Land & Lee, 1994). The authors suggested that the changing optical angular deviation of the tangent or reversal point of a curve from the heading of the vehicle (target drift) is a very good predictor of the curvature of the road ahead.

A number of previous studies reporting on this optical invariant were interpreted to have yielded misleading results due to two methodological reasons. (a) The use of a passive observation paradigm may have led to a lower degree of accuracy than could have been achieved with the use of actively controlled tasks. (b) More importantly, animals evaluate their movement with respect to specific optical discontinuities (Gibson, 1966). Early investigations on heading perception generally asked subjects to simply indicate the direction of self-motion based on a previously viewed path of translation (e.g., Llewellyn, 1971; Johnston et al., 1973). I have suggested that ecologically more valid task demands may have led to considerably higher judgment accuracies (see p. 72). Even the practice of more recent investigations to ask subjects to indicate their heading direction with respect to a nominal target that appears on the screen after all display motion has stopped is questionable in this context (see p. 146). Be that as it may, the present investigation has shown that observers can control their heading in a simulated environment with considerable accuracy (mean heading error at the conclusion of control episodes = 1.2 deg) using target-drift as the only functional type of information.

A final point on this topic: Cutting and colleagues (Cutting, 1986; Cutting et al., 1992; Cutting et al., 1995, Vishton & Cutting, 1995) suggested that the type of simulation used in Experiments 2 to 4 ('dolly-and-pan') simulates retinal as opposed to optical motion. Conversely, Cutting et al. (1992) suggested that an optical event employing a fixed angle between the simulated viewing direction and the direction of self-motion ('dolly') simulates optical as opposed to retinal motion (Experiment 1). I think that this distinction can only be made if a fixation point is provided in the latter simulation method that would allow subjects to hold their gaze independent of the optical motion of the simulated discontinuities on the display. As it stands, a person's eye should always track the motion of a discontinuity in the display (e.g., the target). The result is that what is projected onto the retina is identical to what would have been projected on to it, had a dolly-and-pan simulation been employed where the gaze can be held by the (now) stationary target. As a consequence, I think it makes more sense to simply consider the first simulation technique as one where the optical motion of the target with respect to the frame of the display is nulled, and the second where it is not. Other than that, the information content as well as the optical foundation for perception are identical in the two cases.

II. GLOBAL OPTICAL FLOW VELOCITY

In Experiments 2 and 3 global optical flow velocity (GOFV) was varied. Findings from previous studies did not allow a prediction with regard to the effect of this manipulation. Some of the reviewed studies had observed a beneficial effect, while others had reported the reverse (see p. 53). An intuitive argument led to the prediction that sensitivity to heading information should increase with increasing GOFV, and that this increased sensitivity should optimize in the top end of the range of human bipedal locomotion. Results of both experiments generally supported this prediction, even though for some subjects this optimization was found to occur at lower levels of GOFV than for others.

A recent investigation by Vishton and Cutting (1995) implied that it may be the spatial displacement of optical discontinuities rather than information contained in the velocity field that is responsible for any performance differences found during manipulations of GOFV. An intriguing case study presented by Zihl, von Cramon and Mai (1983) of a woman who exhibited selective disturbance of the perception of object motion as a result of bilateral brain damage may be of relevance here. While displacement perception seemed unaffected, the woman had no perception of the actual movement of objects. For example, when in a room with other people walking around she felt insecure and unwell, because "people were suddenly here or there but I have not seen them moving" (Zihl et al., 1983, p. 315). At the same time no mention was made of any disturbances with regard to the control of self-motion[40]. While this seems to indicate that different brain regions may be responsible for the perception of object- and self-motion, it can also be taken to indicate that displacement information is indeed more important for the perception and control of self-motion than velocity field information. The results of the present investigation are consistent with the former, but not the latter interpretation. Unlike in Vishton and Cutting's study, performance improved with increasing GOFV in the presence of similar displacement information. Both the utilization of an active control paradigm and the longer event durations used in the present investigation were suggested to be responsible for this apparent discrepancy (see p. 101). Contrasting results from studies into human sensitivity to change in altitude during self-motion (cf. Owen, 1990) were interpreted to indicate that people may use different types of information depending on the task demands of a situation.

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[40] I assume here that no such deficit existed. Unfortunately no formal investigation of self-motion perception was reported.

III. DIFFERENTIAL OPTICAL MOTION

The dolly-and-pan simulation employed in Experiment 2 (eliminating target drift) was also used for Experiments 3 and 4, which were designed to evaluate the functionality of *differential optical motion* for the perception and control of heading. Two types of motion parallax, simple motion parallax (SMP, defined to involve the relative optical motion of two optical discontinuities produced by objects at different distances to the moving point of observation), and differential motion parallax (DMP, involving three such discontinuities) were investigated. Results suggested that some variation of objects in depth is necessary to allow goal directed self-motion (see also Hildreth, 1992; Rieger & Toet, 1985)[41]. A ratio indexing this separation in depth (Equation 3, p. 97), initially introduced to account for performance improvements with decreasing distance to the target, was able to account for most of the performance differences between the simulated environments. Most importantly in the present context, with the effect of σ accounted for, performance in environments that made DMP information available was no better than in those that made SMP information available.

It is important to keep in mind that, unlike target drift, σ is not an optical variable. It simply indexes the magnitude of relative optical motion that is available to an observer to acquire information from. To put it differently, it indexes the rate of

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[41] Note that although differential motion between neighbouring optical elements is "both necessary and sufficient for a decomposition based on flow-field information" (Warren & Hannon, 1990, p. 168), it is only necessary for successful target approach if target drift is absent. This is taken to indicate that, as suggested earlier, decomposition itself is not necessary.

an observer to acquire information from. To put it differently, it indexes the rate of change of optical separation between elements, or their motion parallax. It was also shown that in its present from, σ does not take account of the initial amount of optical separation between discontinuities (performance would be expected to deteriorate with increases in this variable). Similarly, the fact that a log transformation of the dependent variables was successful in increasing the explained amount of variation indicated that the gain in performance for a given increase of σ is large when σ is near 0, and small when it is near 1. Future research will attempt to incorporate these issues into a more detailed formulation.

What then is the optical support for the perception and control of target approach? It was established that *target drift* is one candidate. The presence of motion parallax was found to further improve performance. Under conditions where target drift was not available, motion parallax was necessary for successful performance of the task. In this case, assuming that attention is focused on a target that is to be approached, an obvious candidate for control is *the horizontal optical angle enclosed by that target and a neighbouring object* (δ). Let me begin with the simplest case, approach to a target when only one other object is present. If heading error is 0 deg, δ will increase at a certain rate during linear approach. Importantly, *decreasing* δ *during linear self-motion always specifies that the target will be missed.* The required rate of increase depends on (a) the initial magnitude of δ (δ_i), (b) the magnitude of σ, and (c) whether the target is closer to, or further away from, the observer than the other object. In the special case where the target and the object are along the same plane of sight ($\delta_i = 0$ deg) a successful control strategy is to keep δ at 0 deg. On the other hand, if the two objects are equidistant from the moving point of observation ($\sigma = 0$), the rate at which δ increases does not vary significantly with heading error. In addition, in this case δ *will never decrease* (at least as long as the observer does not actually move away from the two objects). In other words, since δ never specifies that the target will be missed, it becomes impossible to successfully perform the task of target approach. This explains the poor performance in the NDM environment (Experiment 4).

Now consider the case where $\delta_i \neq 0$ and $\sigma \neq 0$. Since decreasing δ specifies that the target will be missed, the first rule is to control the path of self-motion such that δ does not decrease. On the other hand, if δ is kept constant and the object is further away than the target, the path of self-motion will initially be to the opposite side of the target, curving towards it until it is reached. The curvature of the approach path will increase with δ_i and decrease with σ. If the object is closer than the target, keeping δ constant is not a successful control strategy. In this case, or to achieve a linear approach path when the object is further away, it is necessary to let δ grow larger. To reiterate, the appropriate rate of change in δ depends on δ_i, on σ, and on whether the target is closer to, or further away from, the observer than the other object. As pointed out before, σ is not an optical variable, and consequently provides no visual information. δ_i on the other hand only provides information about horizontal optical separation, not about distance. In the present context, there are two obvious optical variables that provide the needed distance information: (a) The vertical optical separation between the points where the target and the object meet the ground, and (b) the magnitude and type of change in δ with control actions (for psychophysical evidence of depth perception as a function of motion parallax see Ono, Rivest & Ono, 1986; Rogers & Graham, 1979). For example, if the object is behind and to the right of the target, the ray of light projected by the bottom of the object will be higher in the array than that projected by the bottom of the target. The magnitude of that angle and its location with respect to the horizon provides distance information. Perhaps more importantly, any control action resulting in a change in the direction of self-motion to the left will decrease δ. The relationship between the direction and rate of change in δ and the direction and magnitude of the control action also yields the necessary distance information and specifies σ. Consequently, the required rate of change in δ becomes available.

In the more general case of a cluttered environment containing *n* objects other than the intended target, there are *n* angles δ in the optic array at the point of observation. As shown above, any δ on its own (δ_x), or rather the rate of change in δ_x specifies the direction of self-motion. As is the case when only one other object is present, the approach task is probably easiest if δ_x can be kept at 0 deg. An alternative control strategy would be to compensate for any decreasing δ_x. Yet another alternative is to keep the rate of increase in $\delta_{1\text{-}n}$ (matched for distance) symmetrical about the target. This only works if there are a number of objects at similar distances to the point of observation to the left and the right of the target. This raises the possibility that people are sensitive to some global description of the transformation of $\delta_{1\text{-}n}$.[42] Efforts to derive mathematical descriptions of the transformations of δ_x and $\delta_{1\text{-}n}$ are currently under way (for a formal derivation of δ_x in terms of optical angles available at the moving point of observation see Appendix L).

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[42] For an alternative approach to the detection of heading information from global (retinal) flow see Perrone (1992) and, for a refinement of his model, Perrone and Stone (1994).

Once completed, it will be possible to evaluate performance during simulated target approach where participants have direct control over those optical variables and compare their performance with that achieved using the control system utilized in the present investigation (first-order (velocity) control over the rate of change in direction of the simulated self-motion in rad/s)[43].

IV. CONCLUSION

The present study demonstrates the utility of an active control paradigm for the study of self-motion perception in general, and of heading perception during constant-altitude target approach in particular. Continuous recording of control inputs in relation to the optical information available to the participants allowed the extraction of a number of dependent variables. As a consequence it became possible to take a multivariate approach to data analysis (concurrent analysis of multiple dependent variables yields more stable results than the univariate alternative). Two different simulation techniques were used to show that, in the absence of target drift, differential optical motion (due to at least two objects located at different distances to the moving point of observation) is a necessary condition for accurate perception and control of target approach.

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[43] Warren and Owen (1982) and Owen and Warren (1982) provide a discussion of this approach in the context of aviation research and flight simulation. For its application to the task of helicopter approach to landing see Owen et al. (1994).

Considering the proclaimed importance of active control for the study of self-motion perception, the fact that the viewing direction was not under the direct control of the participants represents a major limitation of the study. The only available alternative at the time of the study was the use of a second manual control device for this purpose (e.g., left hand control for viewing direction, right hand control for direction of self-motion). However, this alternative was rejected since the increased degree of difficulty associated with controlling such a simulation would have yielded additional undesirable performance variation. The use of a (virtual reality) helmet-mounted display would overcome this problem, since the viewing direction is then under the *natural* control of the participant, i.e., when we move around in the real world we also control our viewing direction by simply turning our head. Since the extent to which a target is 'tracked' with the head is directly related to the amount of target drift available, this would make it possible to investigate the extent to which target drift is *actually* used during target approach (results of the present study have shown that target drift *can* be used).

To account for the finding that performance improved with decreasing distance to the target, a ratio indexing the separation of objects in depth was proposed. With the effect of this *separation ratio* accounted for, performance was not affected by the addition of differential motion parallax information to simulations that already carried simple motion parallax information. As a consequence, the *rate of change in horizontal optical separation* between the target and other objects in the display was proposed to be the primary optical support for the perception and control of target approach. The development of mathematical descriptions of this variable will allow further evaluation of its importance for a number of different tasks.

Ultimately, simulation research has important implications for the development of training simulations. Since the amount of optical information in a simulation is directly proportional to its cost, one of the primary functions of such research is the identification of those optical variables that allow a sufficiently high level of both task performance and transfer of training. The results of the present investigation indicate that the simulation of objects at more than one (intermediate) level of σ does not improve performance for the (simulated) task of target approach.

Similar comments can be made with regard to selection and safety. Like previous simulation research, the present study found that people may be differentially sensitive to properties of the optic flow pattern (for a brief review see Owen, 1990). While individual results with respect to optimizing performance at intermediate to high levels of global optical flow velocity were relatively uniform, those with respect to differential optical motion were a little more equivocal. While no one was able to accurately control the simulation in the absence of both target drift and differential optical motion, some of the participants were more sensitive to changes in σ than others. This supports the position that effective training may necessitate selection of trainees with regard to their sensitivity to the optical information present in a simulation (Gibson, 1947).

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APPENDICES

APPENDIX A

Simulation specifications and detailed description of all environments

Eyeheight (h): 500 units

Initial distance to target: 24 h

I. POLYGONS

Sky: Covers the top half of the screen. Colour[1]: RGB 30 00 00 (off white).

Ground plane: Covers the lower half of the screen. Colour: RGB 7f 7f 7f (light gray, one shade darker than sky).

Target: Constructed of twelve diamond shaped polygons, each measuring 100 units (0.2 h) in the vertical diagonal, and 60 units (0.12 h) in the horizontal diagonal. Colour: RGB 00 00 bf (bright yellow). Two polygon groups were created by vertically arranging two sets of six polygons. Subsequently, one group was rotated 90 deg about the vertical axis. When placed at the same position in an environment, the two sets combine to form an object appearing to be constructed of six vertically arranged polygons (independent of the direction from which it is approached).

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[1] All colours are 24-bit.

Objects: Constructed of four diamond shaped, vertically arranged polygons of the same dimensions as the target. Colours: Dark: RGB 18 18 18 (gray, four shades darker than the sky); Medium: RGB 3a 3a 3a (gray, three shades darker than the sky); Light: RGB 4f 4f 4f (gray, two shades darker than the sky).

II. PLACEMENT OF OBJECTS IN EACH ENVIRONMENT

(1) Target-only Environment: Experiments 1 and 2

The Target-only environment consisted of the ground plane, sky, and the target, which was located at a distance of 24 h from the initial viewing position. At the onset of a trial the horizontal optical angle enclosed by the target and the centre of the display was either 0, 2, or 4 deg (Figure A-1, α). Similarly, the initial horizontal optical angle enclosed by the focus of radial outflow (FRO) and the target was either 2, 4, or 6 deg (Figure A-1, β).

(2) Differential Motion Parallax Environment

(a) Experiments 1, 2, and 3. In the differential motion parallax (DMP) environment six rows of objects were added to the control environment, such that three of them appeared on each side of the movement path, parallel to a straight line connecting the initial point of observation and the target (Figure A-2). Objects to the (left) right of that line were rotated (anti)clockwise 20 deg about the vertical axis, thus maximising the visual angles enclosed by them as the simulated point of observation moved past. The colour of the objects was Dark along the two rows closest to the target, Medium along the rows in the middle distance, and Light along the two rows in the far distance.

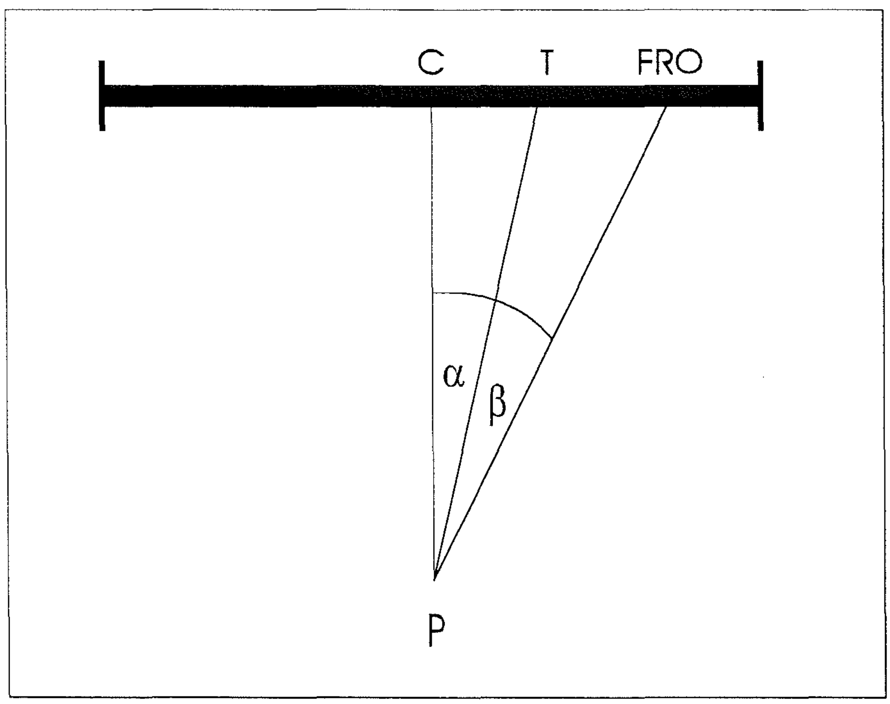

Figure A-1. Initial conditions (bold line = projection surface, P = projection point, C = centre of projection surface, T = target location, FRO = location of focus of radial outflow).

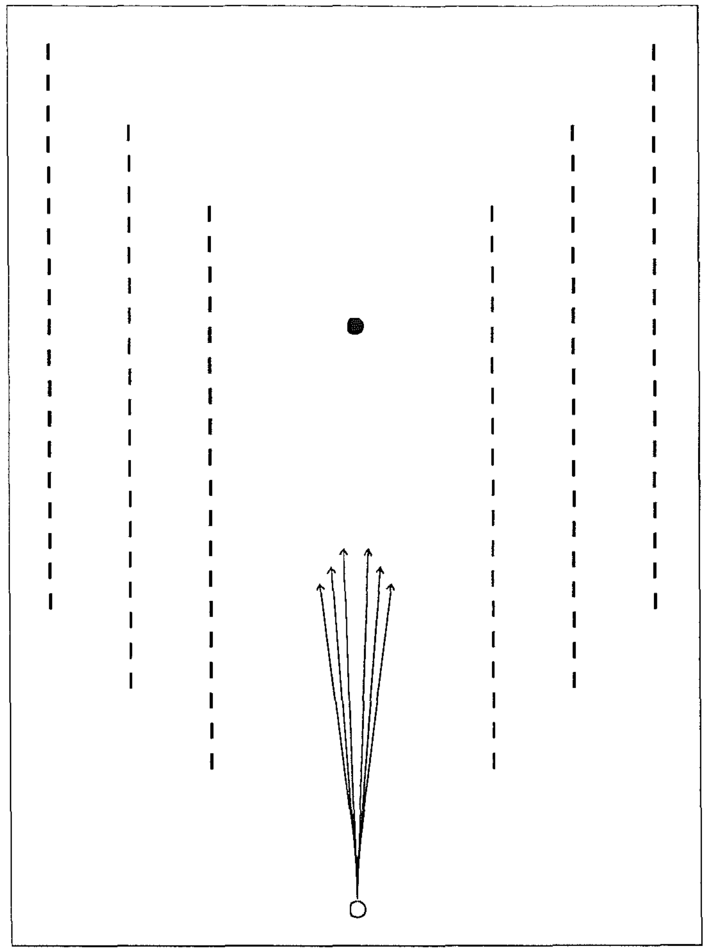

Figure A-2. Schematic representation of the differential motion parallax (DMP) environment viewed from above. The open circle represents the position of the simulated egolocus at trial onset. The filled circle represents the target. The six arrows represent the six possible initial deviations of the instantaneous movement direction from the target. Each of the six dashed lines represents a row of objects in the environment.

The distance between the rows on each side as well as between objects within each row was 4 h, and between the two sets of three rows 8 h (the target was placed halfway between the two sets). The two rows closest to the initial viewing position consisted of 11 objects each (Object 1). The two rows in the middle distance consisted of 16 objects each (Object 2). The two rows in the far distance consisted of

21 objects each (Object 3). This yielded a total of 96 objects (other than the target) in the display.

Overall, this resulted in the following characteristics (Figure A-3): No objects in the simulated environment were placed inside a segment delineated by $\chi = 8$ deg (4 deg on either side of the target) viewed from the initial point of observation. Similarly, no objects were placed inside the segment delineated by the angle $\delta = 16$ deg enclosed by the lines connecting the target with the last objects of the rows on opposite sides of the target. In addition, the angle enclosed by the lines connecting the first object in each row and the target with the initial point of observation (ε) was not less than 22.5 deg.

(b) Experiment 4. In the DMP environments of Experiment 4 three rows of objects were placed perpendicular to the line connecting the initial point of observation and the target. One of the rows was always placed at the location of the target, the other two either behind (DMPB), in front of (DMPF), or to either side of that row (DMPM). Objects to the left (right) of the target were rotated (anti)clockwise 20 deg about the vertical axis, thus maximising the visual angles enclosed by them as the simulated egolocus moved past. Objects other than the target in the row nearest to the initial point of observation were coloured Dark, those in the middle row were coloured Medium, and those in the far row were coloured Light. The rows were 2 h apart, and all objects within a row were spaced 0.4 h apart.

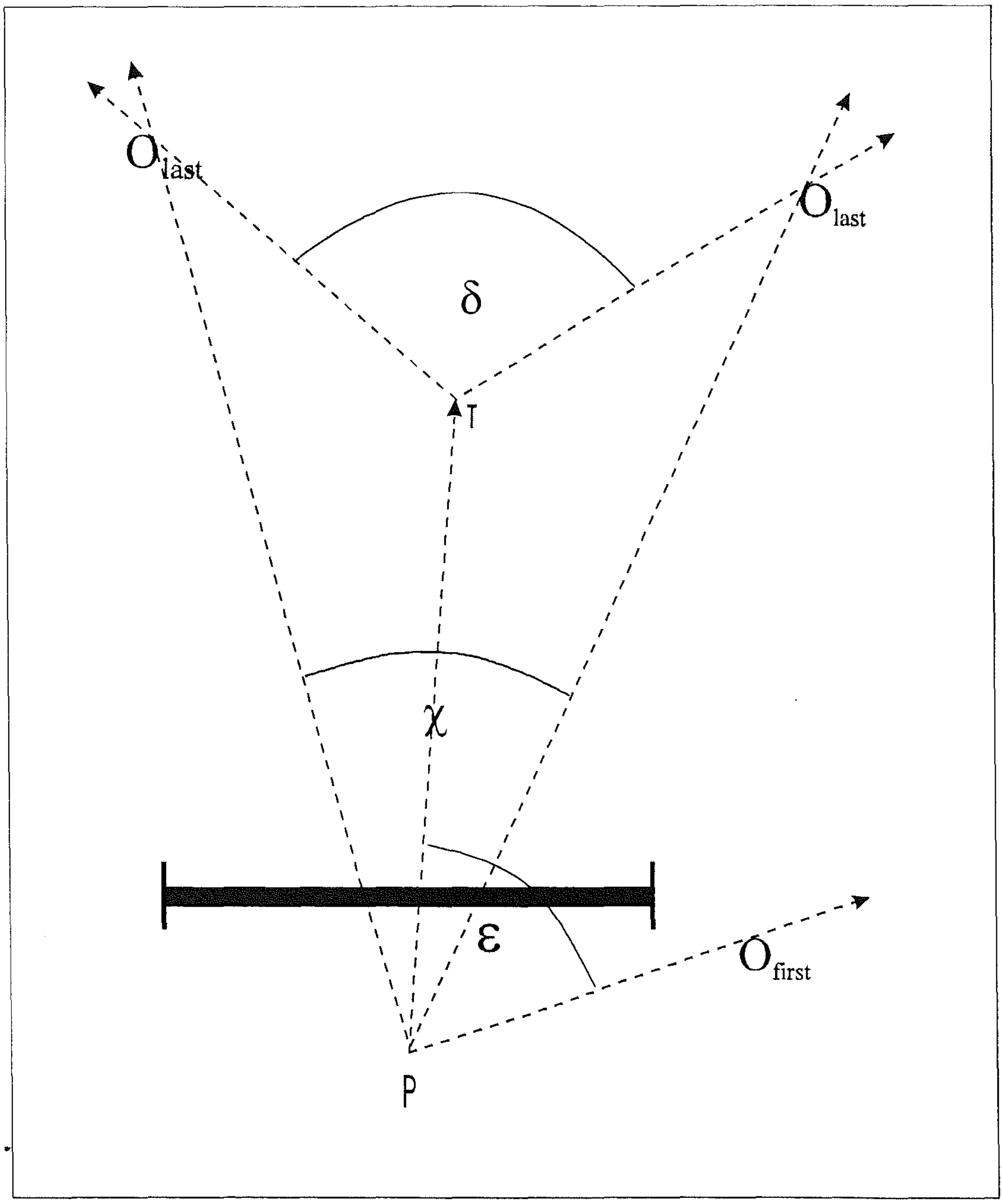

Figure A-3. Specification for simulated environments (bold line = window into simulated environment, P = projection point, T = target location, O = location of first and last objects in a row, δ = 16 deg, χ = 8 deg, ε = 22.5 deg.

(3) Simple Motion Parallax Environment

(a) Experiment 3. The simple motion parallax (SMP) environment retained the basic layout of the DMP environment (Figure A-2). The only change was the removal of the middle row of objects on either side of the target.

(b) Experiment 4. In the SMP environments of Experiment 4 two rows of objects were placed perpendicular to the line connecting the initial point of observation and the target. One of the rows was placed at the location of the target, the other either 4 h behind (SMPB) or 4 h in front of the target (SMPF). Objects to the left (right) of the target were rotated (anti)clockwise 20 deg about the vertical axis, thus maximising the visual angles enclosed by them as the simulated egolocus moved past. Objects other than the target in the row nearer to the initial point of observation were coloured Dark, those in the farther row were coloured Medium. All objects in a row were spaced 0.4 h apart.

(4) Control Environment

(a) Experiment 3. In Experiment 3 the NMP (no motion parallax) environment retained the basic layout of the DMP environment (Figure A-2). However, only one row of objects on each side of the target (the one closest to the target) was retained.

(b) Experiment 4. In the NDM (no differential motion) environment of Experiment 4 one row of objects was placed perpendicular to the line connecting the initial point of observation and the target, at the location of the target. Objects to the left (right) of the target were rotated (anti)clockwise 20 deg about the vertical axis, thus maximising the visual angles enclosed by them as the simulated egolocus moved past. All objects other than the target were coloured Dark, and all objects were spaced 0.4 h apart.

(5) Random Environment: Experiments 1, 2, 3, and 4

In the Random environment 108 objects were added to the Target-only environment. The following procedure yielded what was judged to be an adequate and relatively uniform density of objects: Exact positions were determined randomly with a number of constraints. Beginning at the initial viewing position, facing the target, an imaginary line was extended 4 h to each side. Another line was extended 24 h directly towards the target. Subsequently, a rectangle of 8 x 24 h was constructed. This was then divided into 5 x 15 = 75 squares of 1.6 h on a side. One of Objects 1, 2, or 3 (chosen at random) was then randomly positioned in each of the squares, such that each type of object occupied a third of the squares.

The procedure was then repeated beginning at the target position (facing away from the initial position). This time, an imaginary line was extended 8 h to each side, and 22.4 h ahead. This yielded a rectangle of 16 x 22.4 h, which was then divided into 5 x 7 = 35 squares of 3.2 h on a side. Finally, one object was placed into each of 33 of the squares (Figure A-4). A third of the objects was coloured Dark, a third Medium, and a third Light.

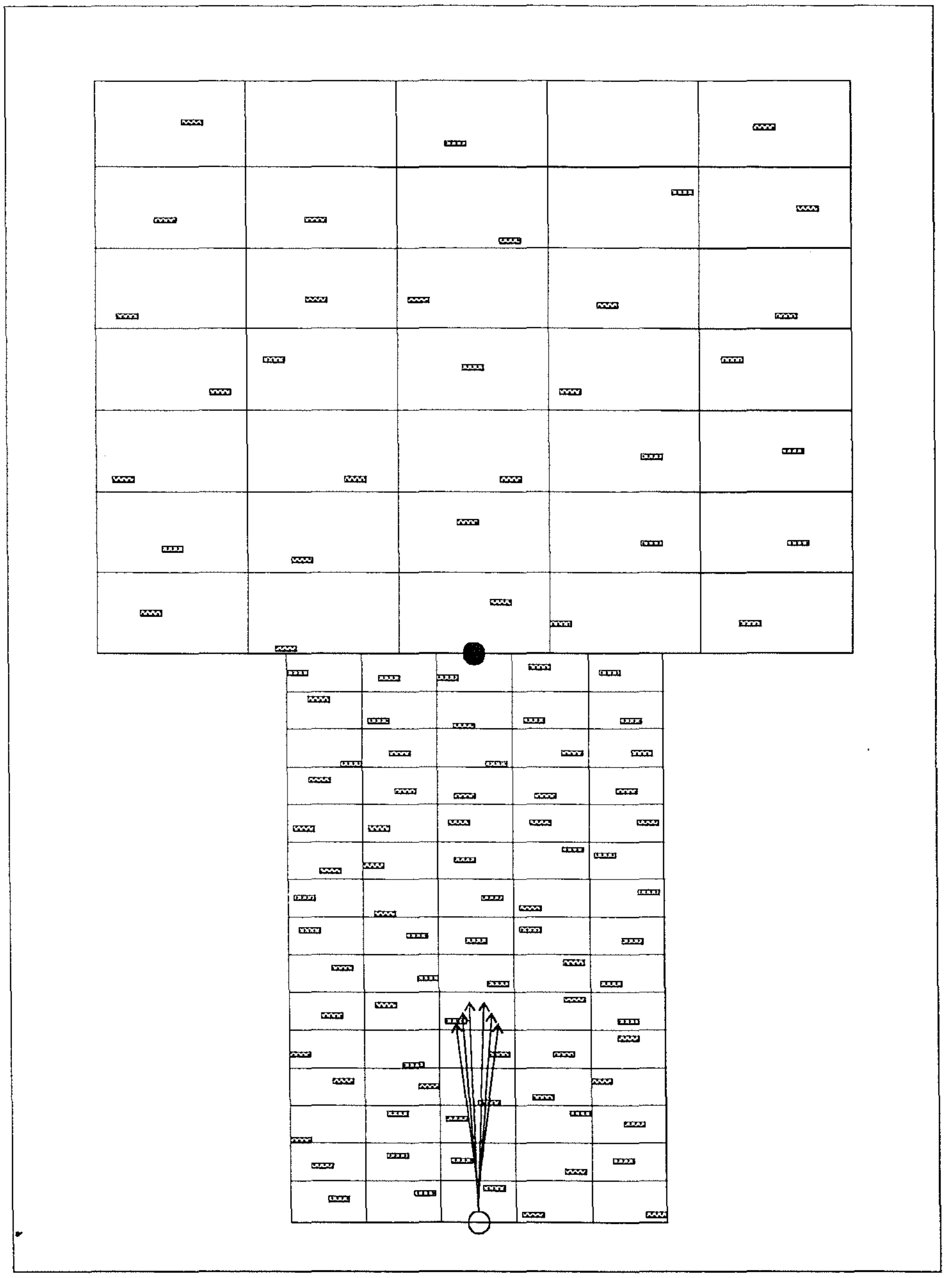

Figure A-4. Schematic representation of the random environment viewed from above. The open circle represents the position of the simulated egolocus at trial onset. The filled in circle represents the target. The filled rectangles represent objects other than the target. One object was randomly placed into 108 of the 110 squares (here represented as open rectangles). The squares were not visible during simulation.

APPENDIX B

Instructions for Experiment 1

Screen 1:

You will be moving forward with constant speed. Using the joystick, you have control over the direction in which you are moving.

Your task is to control the direction in which you are moving such that you reach the target (yellow pole) in the shortest possible time. In other words, you should try to get to the target IN A STRAIGHT LINE.

YOU ONLY MOVE IN A STRAIGHT LINE WHEN YOU LET GO OF THE CONTROL.

Screen 2:

Usually, you are confronted with situations in which you face in the direction in which you are moving. In the following trials you will be facing to one side of the movement direction (at some constant angle). This is similar to walking while keeping your head turned to one side.

Screen 3:

Five demonstration trials will precede six practice trials and 90 experimental trials. Please feel free to ask any questions throughout the demonstration and the practice trials.

Screen 4:

In the first demonstration trial you will be moving towards the target in a straight line while facing in the direction of movement.

Please do not use the joystick during this trial; just observe.

READY FOR DEMONSTRATION TRIAL 1.

Screen 5:

In the next trial you will again be moving towards the target in a straight line. But this time you are facing to the RIGHT of the movement direction.

Please do not use the joystick during this trial.

READY FOR DEMONSTRATION TRIAL 2.

Screen 6:

The next trial is a repeat of the previous one. This time feel free to use the control to get a feel for how it reacts.

READY FOR DEMONSTRATION TRIAL 3.

Screen 7:

In the fourth demonstration trial you will be facing to the LEFT of the movement direction. You will again be moving straight towards the target.

Please do not use the joystick during this trial.

READY FOR DEMONSTRATION TRIAL 4.

Screen 8:

The following trial is a repeat of the previous trial. This time you may use the control again.

READY FOR DEMONSTRATION TRIAL 5.

Screen 9:

The six practice trials will begin as soon as you pull the trigger. They consist of trials taken out of the actual experiment.

REMEMBER TO ENGAGE IN CONTROL ACTIONS ONLY IF YOU ARE NOT SATISFIED WITH YOUR PRESENT HEADING DIRECTION.

READY FOR PRACTICE TRIAL 1.

Screen 10:

You are now ready to begin with the experimental trials. If you have any more questions, now is your last chance to ask them.

Please tell the experimenter when you are ready to start.

APPENDIX C

Balancing procedure for Experiment 1

Events with initial deviations of the target from the centre of the display of -4 or -2 deg were denominated as "L", those with deviations of 2 or 4 deg as "R"[2]. Half of those with a deviation of 0 deg were denominated as "L", the other half as "R". Similarly, initial deviations of the focus of radial outflow (FRO) from the target of -6, -4, or -2 deg were denominated as "L", and those with deviations of 2, 4, and 6 deg were denominated as "R".

This resulted in four possible target / FRO sequences for events, i.e., LL, LR, RL, and RR. Two of those of those four groups (LL and RR) contained 23 events, the other two contained 22 events. Subsequently, 18 blocks containing five events each were constructed by randomly drawing one from each of the groups. In addition, the 18 blocks were manipulated such that:

1. No more than two of the same environments were present in any one block. Consequently, no environment could be presented more than four times in a row.

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[2] Negative signs indicate deviation to the left, positive to the right. Note also, that the term 'environment' refers to the physical layout of the simulated landscape. Consequently, there are three different environments (Target-only, DMP, Random). In contrast, the term "event" denotes a particular set of initial conditions, i.e., in addition to denoting a particular environment, it also refers to a particular initial deviation of both the target from the centre of the display and of the FRO from the target.

2. No more than one event with an initial target deviation of 0 deg from the centre of the display was present in any one block. Consequently, no more than two such events could be presented consecutively.
3. No more than two events with initial target deviations to the same side of the centre of the display were present in any one block. Consequently, no more than four such events could be presented consecutively.
4. No more than three events with initial FRO deviations to the same side of the target were present in any one block. Consequently, no more than six such events could be presented consecutively.
5. No more than two events with identical initial FRO deviations from the target were present in any one block. Consequently, no more than four such events could be presented consecutively.

APPENDIX D

Tables of results for multivariate analyses of variance for Experiment 1

Table D-1. Univariate tests of significance of the interaction between left and right heading error (θ_i) and $FRO_{dev\text{-}C}$ (df = 1, 464).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ|	4.145	297.313	4.145	.641	6.468	.011
Mean |θ| during zero-ctrl	2.994	293.457	2.994	.632	4.734	.030
Mean |θ| at ctrl conclusion	4.369	273.559	4.369	.590	7.410	.007
.....Associated SD	2.257	158.090	2.257	.341	6.624	.010
Mean |θ| at ctrl initiation	6.835	506.080	6.835	1.091	6.267	.013
.....Associated SD	3.980	190.127	3.980	.410	9.713	.002
Reaction time	4.983	707.851	4.983	1.526	3.266	.071
No. correct ctrls	4.789	1965.879	4.789	4.237	1.130	.288
No. incorrect ctrls	.086	172.685	.086	.372	.232	.631

Table D-2. Univariate tests of significance of the main effect of left or right θ_i (df = 1, 428).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ|	0.043	266.104	0.043	0.622	0.069	.793
Mean |θ| during zero-ctrl	0.115	262.203	0.115	0.612	0.188	.665
Mean |θ| at ctrl conclusion	0.111	248.839	0.111	0.581	0.191	.662
.....Associated SD	0.000	147.912	0.000	0.346	0.001	.981
Mean |θ| at ctrl initiation	1.162	450.733	1.162	1.053	1.103	.294
.....Associated SD	0.251	178.180	0.251	0.416	0.603	.438
Reaction time	0.583	588.078	0.583	1.374	0.424	.515
No. correct ctrls	3.521	1851.250	3.521	4.325	0.814	.367
No. incorrect ctrls	0.926	166.463	0.926	0.389	2.381	.124

Table D-3. Univariate tests of significance of the interaction between $T_{dev\text{-}C}$ and $FRO_{dev\text{-}C}$ (df = 1, 428).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ|	29.684	266.104	29.684	0.622	47.744	.000
Mean |θ| during zero-ctrl	27.434	262.203	27.434	0.613	44.782	.000
Mean |θ| at ctrl conclusion	23.865	248.839	23.865	0.581	41.048	.000
.....Associated SD	51.472	450.733	51.472	1.053	48.876	.000
Mean |θ| at ctrl initiation	7.091	147.912	7.091	0.346	20.518	.000
.....Associated SD	9.597	178.180	9.597	0.416	23.052	.000
Reaction time	56.141	588.078	56.141	1.374	40.859	.000
No. correct ctrls	7.002	1851.250	7.002	4.325	1.619	.204
No. incorrect ctrls	2.676	166.463	2.676	0.389	6.880	.009

Table D-4. Univariate tests of significance of the within-cells regression effect for covariates $|T_{dev\text{-}C}|$, $|FRO_{dev\text{-}C}|$, $|\theta_i|$, and trial number (df = 4, 533).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ|	86.798	238.738	21.699	0.448	48.446	.000
Mean |θ| during zero-ctrl	72.257	248.523	18.064	0.466	38.742	.000
Mean |θ| at ctrl conclusion	82.581	215.754	20.645	0.405	51.002	.000
.....Associated SD	35.818	140.795	8.954	0.264	33.898	.000
Mean |θ| at ctrl initiation	147.597	405.506	36.899	0.760	48.501	.000
.....Associated SD	40.760	171.030	10.190	0.321	31.756	.000
Reaction time	191.472	547.143	47.868	1.027	46.631	.000
No. correct ctrls	97.075	2117.647	24.269	3.973	6.108	.000
No. incorrect ctrls	6.868	183.465	1.717	.344	4.988	.001

Table D-5. Univariate tests of the main effect for Environment (df = 2, 533).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean $\|\theta\|$	10.208	238.738	5.104	0.448	11.396	.000
Mean $\|\theta\|$ during zero-ctrl	8.043	248.523	4.021	0.466	8.625	.000
Mean $\|\theta\|$ at ctrl conclusion	8.931	215.754	4.466	0.405	11.032	.000
.....Associated SD	3.507	140.795	1.754	0.264	6.639	.001
Mean $\|\theta\|$ at ctrl initiation	20.214	405.506	10.107	0.761	13.285	.000
.....Associated SD	7.537	171.030	3.768	0.321	11.744	.000
Reaction time	35.003	547.143	17.501	1.027	17.049	.000
No. correct ctrls	56.313	2117.647	28.157	3.973	7.087	.001
No. incorrect ctrls	.192	183.465	.096	.344	.279	.757

Table D-6. Table of means for the three environments.

Variable	Target-only environment	DMP environment	Random environment
Mean $\|\theta\|$	1.313	1.028	1.015
Mean $\|\theta\|$ during zero-ctrl	1.218	.955	.964
Mean $\|\theta\|$ at ctrl conclusion	1.203	.940	.920
.....Associated SD	.904	.770	.711
Mean $\|\theta\|$ at ctrl initiation	1.896	1.489	1.483
.....Associated SD	1.090	.904	.806
Reaction time (s)	2.050	1.626	1.442
No. of correct ctrl episodes	5.861	5.636	5.092
No. of incorrect ctrl episodes	.244	.2887	.278

APPENDIX E

Instructions for Experiment 2

Screen 1:

You will be moving forward with constant speed. Using the joystick, you have control over the direction in which you are moving.

Your task is to control the direction in which you are moving such that you reach the target (yellow pole) in the shortest possible time. In other words, you should try to get to the target IN A STRAIGHT LINE.

YOU ONLY MOVE IN A STRAIGHT LINE WHEN YOU LET GO OF THE CONTROL.

Screen 2:

Usually, you are confronted with situations in which you face in the direction in which you are moving.

In the following trials you will ALWAYS BE FACING THE TARGET, even though you may not be moving directly towards it

Screen 3:

Four demonstration trials will precede six practice trials and 54 experimental trials. Please feel free to ask any questions throughout the demonstration and the practice trials.

Screen 4:

In the first demonstration trial you will be moving in a straight line towards a point to the RIGHT of the target.

Please do not use the joystick during this trial; just observe.

READY FOR DEMONSTRATION TRIAL 1.

Screen 5:

The next trial is a repeat of the previous one. This time feel free to use the control to get a feel for how it reacts.

READY FOR DEMONSTRATION TRIAL 2.

Screen 6:

In the third demonstration trial you will be moving in a straight line towards a point to the LEFT of the target.

Please do not use the joystick during this trial.

READY FOR DEMONSTRATION TRIAL 3.

Screen 7:

The following trial is a repeat of the previous trial. This time you may use the control again.

READY FOR DEMONSTRATION TRIAL 4.

Screen 8:

The six practice trials will begin as soon as you pull the trigger. They consist of trials taken from the actual experiment.

REMEMBER TO ENGAGE IN CONTROL ACTIONS ONLY IF YOU ARE NOT SATISFIED WITH YOUR PRESENT HEADING DIRECTION.

READY FOR PRACTICE TRIAL 1.

Screen 10:

You are now ready to begin with the experimental trials. If you have any more questions, now is your last chance to ask them.

Please tell the experimenter when you are ready to start.

APPENDIX F

Balancing procedure for Experiment 2

Events were randomly selected to form 18 blocks containing three events each.

In addition, the 18 blocks were manipulated such that:

1. No more than two of the same environments were present in any one block. Consequently, no environment could be presented more than four times in a row.
2. No more than two events with initial FRO deviations to the same side of the target were present in any one block. Consequently, no more than four such events could be presented consecutively.
3. No more than one event with a given initial FRO deviation from the target was present in any one block. Consequently, no more than two such events could be presented consecutively.
4. No more than two events with a given global optical flow velocity were present in any one block. Consequently, no more than four such events could be presented consecutively.

For final presentation of trials, both events within blocks as well as order of block presentation were randomized.

APPENDIX G

Tables of results for multivariate analyses of variance for Experiment 2

Table G-1. Univariate tests of the main effect for Environment (df = 1, 208).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean $\|\theta\|$	21.372	254.439	21.372	1.223	17.472	.000
Mean $\|\theta\|$ during zero-ctrl	22.507	268.840	22.507	1.293	17.414	.000
Mean $\|\theta\|$ at ctrl conclusion	15.651	264.588	15.651	1.272	12.303	001
.....Associated SD	1.910	138.506	1.910	0.666	2.869	.092
Mean $\|\theta\|$ at ctrl initiation	44.353	517.920	44.353	2.490	17.812	.000
.....Associated SD	1.979	138.002	1.979	0.663	2.983	.086
Reaction time	201.796	2255.159	201.796	10.842	18.612	.000
No. correct ctrls	3.847	852.908	3.847	4.101	0.938	.334
No. incorrect ctrls	0.000	39.912	0.000	0.192	0.000	.987

Table G-2. Univariate tests of significance of the main effect for global optical flow velocity (df = 2, 208).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean $\|\theta\|$	9.408	254.439	4.704	1.223	3.845	.023
Mean $\|\theta\|$ during zero-ctrl	10.237	268.840	5.118	1.293	3.960	.021
Mean $\|\theta\|$ at ctrl conclusion	15.844	264.588	7.922	1.272	6.228	.002
.....Associated SD	16.716	138.506	8.358	0.666	12.551	.000
Mean $\|\theta\|$ at ctrl initiation	23.096	517.920	11.548	2.490	4.638	.011
.....Associated SD	25.529	138.002	12.765	0.663	19.239	.000
Reaction time	452.913	2255.159	226.456	10.842	20.887	.000
No. correct ctrls	170.662	852.908	85.331	4.101	20.810	.000
No. incorrect ctrls	3.734	39.912	1.867	0.192	9.730	.000

Table G-3. Univariate tests of significance of the within-cells regression effect (df = 2, 570).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ|	27.170	422.538	13.585	0.741	18.326	.000
Mean |θ| during zero-ctrl	20.533	441.320	10.266	0.774	13.260	.000
Mean |θ| at ctrl conclusion	25.291	424.241	12.646	0.744	16.990	.000
.....Associated SD	12.037	248.603	6.018	0.436	13.799	.000
Mean |θ| at ctrl initiation	43.610	827.587	21.805	1.452	15.018	.000
.....Associated SD	15.714	271.556	7.857	0.476	16.492	.000
Reaction time	176.479	3284.172	88.240	5.762	15.315	.000
No. correct ctrls	41.420	2341.432	20.710	4.108	5.042	.007
No. incorrect ctrls	0.354	160.939	0.177	0.282	0.626	.535

Table G-4. Univariate tests of significance of the interaction between environment and simulation type (df = 1, 570).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ|	12.828	422.538	12.828	0.741	17.304	.000
Mean |θ| during zero-ctrl	14.478	441.320	14.478	0.774	18.700	.000
Mean |θ| at ctrl conclusion	9.043	424.241	9.0428	0.744	12.150	.001
.....Associated SD	0.557	248.603	0.557	0.436	1.276	.259
Mean |θ| at ctrl initiation	27.307	827.587	27.307	1.452	18.808	.000
.....Associated SD	0.295	271.556	0.295	0.476	0.619	.432
Reaction time	103.470	3284.172	103.470	5.762	17.958	.000
No. correct ctrls	22.036	2341.432	22.036	4.108	5.365	.021
No. incorrect ctrls	0.004	160.939	0.004	0.282	0.014	.905

APPENDIX H

Balancing procedure for Experiment 3

Events were randomly selected to form 18 blocks containing four events each. In addition, the 18 blocks were manipulated such that:

1. No more than two of the same environments were present in any one block. Consequently, no environment could be presented more than four times in a row.
2. No more than one event with a given initial FRO deviation from the target was present in any one block. Consequently, no more than two such events could be presented consecutively.
3. No more than two events with identical global optical flow velocities were present in any one block. Consequently, no more than four such events could be presented consecutively.

For final presentation of trials, both events within blocks as well as order of block presentation were randomized.

APPENDIX I

Tables of results for multivariate analyses of variance for Experiment 3

Table I-1. Univariate tests of significance of the within-cells regression effect for the covariates $|\theta_i|$ and trial number (df = 2, 850).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean $\|\theta\|$	26.315	509.699	13.157	0.600	21.942	.000
Mean $\|\theta\|$ during zero-ctrl	18.995	542.487	9.498	0.638	14.881	.000
Mean $\|\theta\|$ at ctrl conclusion	30.355	407.201	15.177	0.479	31.681	.000
.....Associated SD	19.170	257.911	9.585	0.303	31.590	.000
Mean $\|\theta\|$ at ctrl initiation	50.931	838.165	25.465	0.986	25.825	.000
.....Associated SD	25.850	389.447	12.925	0.458	28.210	.000
Reaction time	464.795	5261.341	232.397	6.190	37.545	.000
No. correct ctrls	66.794	2900.193	33.397	3.412	9.788	.000
No. incorrect ctrls	0.315	241.922	0.157	0.285	0.553	.576

Table I-2. Univariate tests of significance of the main effect for Environment (df = 3, 850).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean $\|\theta\|$	27.937	509.699	9.312	0.600	15.530	.000
Mean $\|\theta\|$ during zero-ctrl	31.814	542.487	10.605	0.638	16.616	.000
Mean $\|\theta\|$ at ctrl conclusion	13.885	407.201	4.628	0.479	9.661	.000
.....Associated SD	4.769	257.911	1.590	0.303	5.240	.001
Mean $\|\theta\|$ at ctrl initiation	34.111	838.165	11.370	0.986	11.531	.000
.....Associated SD	9.997	389.447	3.332	0.458	7.273	.000
Reaction time	216.594	5261.341	72.198	6.190	11.664	.000
No. correct ctrls	5.504	2900.193	1.835	3.412	0.538	.657
No. incorrect ctrls	1.608	241.922	0.536	0.284	1.883	.131

Table I-3. Means for the four environments.

	Environment			
Variable	NMP	SMP	DMP	Random
Mean $\|\theta\|$	1.371	1.242	1.232	0.879
Mean $\|\theta\|$ during zero-ctrl	1.361	1.221	1.196	0.834
Mean $\|\theta\|$ at ctrl conclusion	1.096	1.015	1.048	0.759
.....Associated SD	0.769	0.715	0.757	0.574
Mean $\|\theta\|$ at ctrl initiation	1.928	1.797	1.820	1.390
.....Associated SD	0.999	0.942	0.950	0.715
Reaction time (s)	2.755	2.465	2.303	1.423
No. of correct ctrl episodes	4.907	4.722	4.778	4.690
No. of incorrect ctrl episodes	0.338	0.245	0.250	0.227

Table I-4. Univariate tests of the main effect for global optical flow velocity (df = 2, 850).

Variable	**Hypoth. SS**	**Error SS**	**Hypoth. MS**	**Error MS**	**F**	**p**
Mean $\|\theta\|$	7.689	509.699	3.844	0.600	6.411	.002
Mean $\|\theta\|$ during zero-ctrl	7.458	542.487	3.729	0.638	5.843	.003
Mean $\|\theta\|$ at ctrl conclusion	9.469	407.201	4.734	0.479	9.882	.000
.....Associated SD	16.502	257.911	8.251	0.303	27.192	.000
Mean $\|\theta\|$ at ctrl initiation	28.746	838.165	14.373	0.986	14.576	.000
.....Associated SD	34.634	389.447	17.317	0.458	37.796	.000
Reaction time	1253.645	5261.341	626.823	6.190	101.267	.000
No. correct ctrls	689.559	2900.193	344.779	3.412	101.049	.000
No. incorrect ctrls	14.605	241.922	7.302	0.285	25.657	.000

APPENDIX J

Balancing procedure for Experiment 4

Events were randomly selected to form 21 blocks containing four events each. In addition, the 21 blocks were manipulated such that:

1. No more than two of the same environments were present in any one block. Consequently, no environment could be presented more than four times in a row.
2. No more than one event with a given initial FRO deviation from the target was present in any one block. Consequently, no more than two such events could be presented consecutively.
3. No more than two events with a given field of view were present in any one block. Consequently, no more than four such events could be presented consecutively.

For final presentation of trials both events within blocks as well as order of block presentation were randomized.

APPENDIX K

Tables of results for multivariate analyses of variance for Experiment 4

Table K-1. Univariate tests of significance of within cells regression effect for covariates $|\theta_i|$ and trial number (df = 2, 908). Analysis includes data from the NDM environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean $|\theta|$ (deg)	291.112	6010.575	145.556	6.620	21.989	.000
Mean $|\theta|$ during zero-ctrl	249.381	5064.137	124.690	5.577	22.357	.000
Mean $|\theta|$ at ctrl conclusion	224.059	19839.230	112.030	21.849	5.127	.006
.....Associated SD	28.913	8192.132	14.456	9.022	1.602	.202
Mean $|\theta|$ at ctrl initiation	619.575	15176.810	309.787	16.715	18.534	.000
.....Associated SD	76.576	8210.313	38.288	9.042	4.234	.015
Reaction time (s)	173.473	7257.116	86.737	7.992	10.852	.000

Table K-2. Univariate tests of significance of within cells regression effect for covariates $|\theta_i|$ and trial number (df = 2, 778). Analysis excludes data from the NDM environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean $|\theta|$ (deg)	50.678	686.075	25.339	0.882	28.734	.000
Mean $|\theta|$ during zero-ctrl	47.028	710.757	23.514	0.914	25.739	.000
Mean $|\theta|$ at ctrl conclusion	26.959	936.489	13.480	1.204	11.198	.000
.....Associated SD	11.165	938.702	5.583	1.207	4.627	.010
Mean $|\theta|$ at ctrl initiation	66.042	1252.772	33.021	1.610	20.507	.000
.....Associated SD	30.440	678.443	15.220	0.872	17.453	.000
Reaction time (s)	291.685	1751.283	145.843	2.251	64.790	.000

table K-3. Univariate tests of main effect for Environment (df = 5, 778). Analysis excludes data from the NDM environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ| (deg)	133.126	686.075	26.625	0.882	30.193	.000
Mean |θ| during zero-ctrl	128.075	710.757	25.615	0.914	28.038	.000
Mean |θ| at ctrl conclusion	89.670	936.489	17.934	1.204	14.899	.000
.....Associated SD	43.524	938.702	8.705	1.207	7.215	.000
Mean |θ| at ctrl initiation	174.997	1252.772	34.999	1.610	21.735	.000
.....Associated SD	64.932	678.443	12.986	0.872	14.892	.000
Reaction time (s)	480.974	1751.283	96.195	2.251	42.734	.000

Table K-4. Univariate tests of differences between environments SMPF and SMPB (df = 1, 255).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ| (deg)	18.146	497.864	18.146	1.952	9.294	.003
Mean |θ| during zero-ctrl	22.269	528.654	22.269	2.073	10.742	.001
Mean |θ| at ctrl conclusion	9.849	540.972	9.849	2.121	4.642	.032
.....Associated SD	1.085	302.170	1.085	1.185	0.915	.340
Mean |θ| at ctrl initiation	21.684	785.495	21.684	3.080	7.039	.008
.....Associated SD	8.148	306.617	8.148	1.202	6.776	.010
Reaction time (s)	44.480	989.984	44.480	3.882	11.457	.001

Table K-5. Univariate tests of differences between environments DMPF, DMPM, and DMPB (df = 2, 368).

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ| (deg)	15.119	312.320	7.560	0.849	8.907	.000
Mean |θ| during zero-ctrl	14.020	323.673	7.010	0.880	7.970	.000
Mean |θ| at ctrl conclusion	9.613	516.146	4.807	1.403	3.427	.034
.....Associated SD	5.642	398.514	2.821	1.083	2.605	.075
Mean |θ| at ctrl initiation	40.048	751.542	20.024	2.042	9.805	.000
.....Associated SD	10.487	365.066	5.244	0.992	5.286	.005
Reaction time (s)	71.131	556.624	35.565	1.513	23.513	.000

Table K-6. Univariate tests of main effect for Type of Information (df = 1, 654). Analysis excludes data from the NDM environment and the Random environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ| (deg)	72.075	857.436	72.075	1.370	52.621	.000
Mean |θ| during zero-ctrl	75.686	900.966	75.686	1.439	52.588	.000
Mean |θ| at ctrl conclusion	29.021	1079.765	29.021	1.725	16.825	.000
.....Associated SD	2.947	711.138	2.947	1.136	2.594	.108
Mean |θ| at ctrl initiation	86.762	1617.652	86.762	2.584	33.575	.000
.....Associated SD	7.678	696.141	7.678	1.112	6.904	.009
Reaction time (s)	149.329	1661.033	149.329	2.653	56.278	.000

Table K-7. Univariate tests of the within-cells regression effect (df = 3, 915). Analysis includes data from the NDM environment and the Random environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ| (deg)	170.325	3225.039	56.775	3.943	14.400	.000
Mean |θ| during zero-ctrl	175.382	3696.722	58.461	4.519	12.936	.000
Mean |θ| at ctrl conclusion	105.513	5236.380	35.171	6.401	5.494	.001
.....Associated SD	44.077	1483.720	14.692	1.814	8.100	.000
Mean |θ| at ctrl initiation	272.884	3859.493	90.961	4.718	19.279	.000
.....Associated SD	44.955	2898.848	14.985	3.544	4.229	.006
Reaction time (s)	436.616	3515.155	145.539	4.297	33.868	.000

Table K-8. Univariate tests of the main effect for Type of Information (df = 2, 915). Analysis includes data from the NDM environment and the Random environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ| (deg)	1105.969	3225.039	552.985	3.943	140.259	.000
Mean |θ| during zero-ctrl	1115.550	3696.722	557.775	4.519	123.423	.000
Mean |θ| at ctrl conclusion	1546.942	5236.380	773.471	6.401	120.828	.000
.....Associated SD	34.797	1483.720	17.399	1.814	9.592	.000
Mean |θ| at ctrl initiation	752.884	3859.493	376.442	4.718	79.785	.000
.....Associated SD	168.044	2898.848	84.022	3.544	23.709	.000
Reaction time (s)	622.366	3515.155	311.183	4.297	72.414	.000

Table K-9. Univariate tests of the within-cells regression effect (df = 3, 653). Analysis excludes data from the NDM environment and the Random environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ|	99.575	828.016	33.192	1.325	25.054	.000
Mean |θ| during zero-ctrl	98.226	869.689	32.742	1.392	23.530	.000
Mean |θ| at ctrl conclusion	59.418	1062.719	19.806	1.700	11.648	.000
.....Associated SD	23.767	705.182	7.922	1.128	7.022	.000
Mean |θ| at ctrl initiation	184.921	1558.181	61.641	2.493	24.725	.000
.....Associated SD	32.154	679.394	10.718	1.087	9.860	.000
Reaction time	433.138	1545.877	144.379	2.473	58.373	.000

Table K-10. Univariate tests of the main effect for Type of Information (df = 1, 653). Analysis excludes data from the NDM environment and the Random environment.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	p
Mean |θ| (deg)	0.050	828.016	0.050	1.325	0.038	.846
Mean |θ| during zero-ctrl	0.067	869.689	0.067	1.391	0.048	.827
Mean |θ| at ctrl conclusion	0.531	1062.719	0.531	1.700	0.312	.576
.....Associated SD	1.424	705.182	1.424	1.128	1.262	.262
Mean |θ| at ctrl initiation	3.062	1558.181	3.062	2.493	1.228	.268
.....Associated SD	4.231	679.394	4.231	1.087	3.892	.049
Reaction time (s)	7.953	1545.877	7.953	2.473	3.216	.073

APPENDIX L

Formal derivation of δ in terms of optical angles available at the moving point of observation

Figure L-1 represents the general case of level linear self-motion toward a target. O is the point of observation moving toward the target at a constant eyeheight. The line OL is the perpendicular to the surface (eyeheight). The two vertical bars represent the target and a second object.

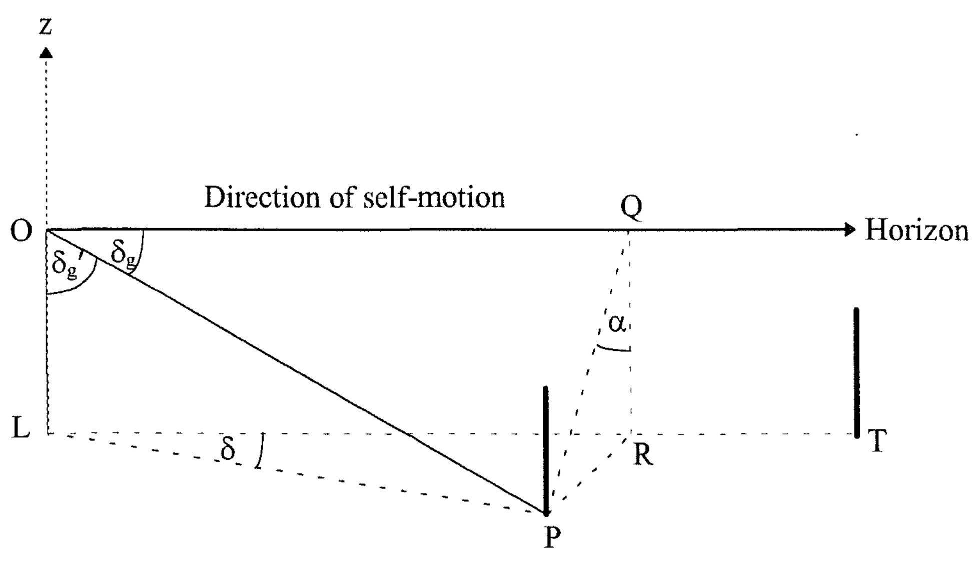

Figure L-1. A moving point of observation with respect to two objects on a surface. The two angles enclosed by solid lines are directly available at the point of observation (O).

P is any point on the surface where the second object meets the surface. T is the point where the target meets the surface. QP and QR are perpendiculars to the line of locomotion which determine angle α, and PR is perpendicular to LT. The angle δ_g is the angular separation of the light rays reflected from P and the horizon in the direction of T (or from P and the intersection of the horizon and a target that is taller than OL). Similarly, the angle δ_g' is the angular separation of the light rays reflected from P and L (alternatively, δ_g' is described by the angular separation of the light rays reflected from the horizon (in the direction of P) and P, less 90 deg). Note that α remains constant as O approaches the target. Consequently, the apparent expansion of the ground at point P will have the magnitude $d\delta / dt$ and will be along the direction of constant α (Gibson, Olum & Rosenblatt, 1954). The angle of interest is δ, the horizontal optical separation of target and object. It can be derived by

$$\delta = \sin^2 (PR/LP), \tag{1}$$

where

$$PR = (\tan \alpha)\, OL \tag{2}$$

and

$$LP = (\tan \delta_g')\, OL. \tag{3}$$

Substituting Equations (2) and (3) into Equation (1) gives

$$\delta = \sin^2 (\tan \alpha / \tan \delta_g'), \tag{4}$$

where

$$\alpha = \cos^2 (OL / QP), \tag{5}$$

$$QP = (\sin \delta_g)\, OP, \tag{6}$$

and

$$OP = OL / \cos \delta_g'. \tag{7}$$

Substituting Equation (7) into Equation (6) gives

$$QP = OL\ (\sin \delta_g) / \cos \delta_g'. \tag{8}$$

Substituting Equation (8) into Equation (5) gives

$$\alpha = \cos^2 (\cos \delta_g' / \sin \delta_g). \tag{9}$$

Finally, substituting Equation (9) into Equation (4) gives

$$\delta = \sin^2 \{\tan[\cos^2 (\cos \delta_g' / \sin \delta_g]\} / \tan \delta_g'. \tag{10}$$

Equation 10 fulfils the requirements of expressing δ in terms of the optical angles available at the moving point of observation. Note that if $\delta_g' + \delta_g = 90$ deg, the light rays reflected off the target and the object occupy the same vertical sector of the array, i.e., $\delta = 0$ deg, and target drift is 0 during self-motion. Similarly, if Equation 9 remains constant over time, continuing self-motion leads straight to the target.